2. Stellar atmospheres: Structure

2.1. Assumptions

- Plane-parallel geometry
- Hydrostatic equilibrium, i.e.
  - no large-scale accelerations comparable to surface gravity
  - no dynamically significant mass loss
- Homogeneity, i.e.
  - no granulation, starspots, prominences, etc.
- No magnetic fields
- LTE in most cases, so
  - excitation, ionization, source function, and thermal velocity distribution are all described by temperature only

Thus, the structure of such a stellar atmosphere is essentially described by temperature and pressure distribution as functions of optical depth.

2.2. Hydrostatic equation

The pressure distribution is established from the assumption on hydrostatic equilibrium. This is illustrated in Fig. 9.1.

The difference in pressure between the top and the bottom of an elemental volume is the weight of the mass per unit area:

\[ dF = \rho gdA dx \]

\[ dP = \frac{dF}{dA} = \rho gdx \]

With \( x \) increasing inward as the pressure does, no negative sign appears in this relation.

Transforming this into the optical depth scale with

\[ d\tau_v = \alpha_v dx = k_v \rho dx \]

we obtain the desired hydrostatic equation:

\[ \frac{dP}{d\tau_v} = \frac{g}{k_v} \]

Fig. 9.1. The force \( F \) is exerted by the overlying gas on the area \( dA \) to give a pressure \( P \). The weight of the material in the volume \( dx dA \) adds a force \( dF \) in the depth \( dx \) increasing the pressure by \( dP \). From Gray (1992).
Here, the pressure is the total pressure supporting the volume. In general, this includes:

- gas pressure, $P_g$
- radiation pressure (in very hot stars), $P_R = \frac{4\sigma}{3c} T^4 = 2.52 \cdot 10^{-15} T^4$ [dyn/cm$^2$]
- magnetic pressure, $P_m = \frac{B^2}{8\pi}$
- turbulence pressure, $P_t = \frac{1}{2} \rho v^2$

Table 9.1 gives some examples: the magnetic field and turbulent velocity values are calculated so that the pressure provided by them equal $P_g$.

The equation of the hydrostatic equilibrium is solved iteratively, usually on logarithmic optical-depth scale. The result depends on the opacity, which is defined by temperature and electronic pressure. Thus, these functions have to be known too.

### 2.3. Temperature distribution

#### In the Sun

The temperature distribution on the Sun (and some other resolved stars) can be deduced from

- limb-darkening
- wavelength dependence of absorption coefficient

The emergent intensity measured on the solar disk:

$$I_v(0, \mu) = \int_0^\infty S_v(\tau_v) e^{-\tau_v/\mu} d\tau_v,$$
where the exponential extinction varies as $\tau_v/\mu$, so the position of the unit optical depth along the line of sight moves upward as the line of sight is shifted from the disk center to the limb.

For a linear source function (in Milne-Eddington approximation):

$$S_v = a + b\tau_v$$

one gets

$$I_v(0,\mu) = a + b\mu,$$

which means that at $\tau_v = \mu$, the specific intensity on the surface at the position $\mu$ equals the source function at the depth $\tau_v$ (Eddington-Barbier relation).

Thus, measurement of intensity across the solar disk can be used to obtain the depth dependence of the source function (Fig. 9.2). Alternatively, source function can be varied until the proper limb-darkening is attained.

![Diagram](image)

**Fig. 9.2.** (a) The limb-darkening observations of Pierce and Waddell (1961) are shown for several wavelengths. The 4000 Å curve is nearly linear. (b) A schematic illustration of the cause of limb darkening is shown. The top and bottom of the photosphere are indicated by the outer and inner circles. Penetration of our line of sight to unit optical depth, as indicated by the heavy line segments, corresponds to different depths in the photosphere depending on $\theta$. The dashed curve indicates the surface of penetration to unit optical depth. Radiation seen at the disk position $\theta_1$ is characteristic of the higher cooler layers than the radiation seen at the disk position $\theta_2$.

The function under the integral (integrand) $S_v(\tau_v)e^{-\tau_v/\mu}$ introduces the concept of formation depth (Fig. 9.3).

Once the source function is found from the surface intensity distribution, it is set equal to the Planck function, and the temperature as a function of depth is obtained (Fig. 9.4).
Fig. 9.3. The depth of formation at three limb distances shows quantitatively the decrease in intensity (area under the curve) and the shift to higher photospheric layers for lines of sight closer to the limbs. Adapted from Pierce and Waddell (1961).

Fig. 9.4. Several solar temperature distributions are compared.
The second source of information on the temperature distribution is the absorption coefficient.

- In the continuum, we see deepest into the solar atmosphere near 1.6 µm. Towards shorter wavelengths, the opacity increases, and at about 0.2 µm the depth of formation is at the temperature minimum. In the chromosphere and higher, the opacity continues to increase.
- In strong spectral lines, the variation of opacity across the line profile provides a similar information. Non-LTE effects are essential here. A combination with limb-darkening measured at different wavelengths produces the best result (Fig. 9.4).

**In other stars**

Only for some stars limb darkening was measured using interferometry. So, in general, the temperature distribution for unresolved stars is determined by assuming the radiative equilibrium, i.e. flux constancy with the depth:

\[
\frac{d}{dx} F(x) = 0,
\]

where flux is integrated over all frequencies. This requirement is achieved iteratively, by varying the source function with depth. The problem is complicated by including the line opacity, but the result is leads to more realistic temperature distribution (Fig. 9.5):

- increase of \( T \) in deeper layers
- decrease of \( T \) in upper layers

Therefore, this effect is referred as “line blanketing”.

![Fig. 9.5. Line blanketing raises the temperature deeper in the photosphere and lowers it in the outer regions. This model has \( T_{\text{eff}} = 6500 \) K, \( \log g = 10^4 \) cm/s², and solar abundances. Data are from Strom and Kurucz (1966).](image-url)
Treatment of convection in the flux constancy method is another difficulty. A convective flux should be included when the temperature gradient exceeds the adiabatic gradient, and the total flux should be conserved. Large uncertainties are found in deep layers (Fig. 9.6).

![Graph showing convection in stellar atmospheres](image)

**Fig. 9.6.** Convection lowers the model's temperatures in the deep layers. The 'radiative' model has no convection. The others are labeled with the ratio of mixing length to pressure scale height. Little light escapes from the layers below \( \log \tau_0 \approx 0.5 \). Data are from Carbon and Gingerich (1969).

### 2.4. Electron pressure

The number of electrons in the stellar atmosphere depends on its chemical composition and temperature, which define the ionization degree:

\[
\frac{N_{ij}}{N_{0j}} = \frac{\Phi_j(T)}{P_e},
\]

where \( \Phi_j(T) \) is constant for a given element and ionization state.

If double ionizations are neglected, the number of electrons contributed by the element \( j \) is

\[
N_{ij} = N_{ej}, \text{ and then}
\]

\[
\frac{\Phi_j(T)}{P_e} = \frac{N_{ej}}{N_{0j}} = \frac{N_{ej}}{N_j - N_{ej}},
\]
where $N_j$ is the total number of $j$-th element particles.

So, we obtain:

$$N_{ej} = N_j \frac{\Phi_j(T)/P_e}{1 + \Phi_j(T)/P_e}$$

And for the pressure

$$\frac{P_e}{P_g} = \frac{\sum N_{ej}kT}{\sum (N_{ej} + N_j)kT} = \frac{\sum N_j[\Phi_j(T)/P_e[1 + \Phi_j(T)/P_e]]}{\sum N_j[(1 + \Phi_j(T)/P_e[1 + \Phi_j(T)/P_e])]$$

If we introduce the number abundance of the element with respect to the number of hydrogen particles per unit volume:

$$A_j = \frac{N_j}{N_H}$$

the electron pressure is then

$$P_e = P_g \frac{\sum A_j[\Phi_j(T)/P_e[1 + \Phi_j(T)/P_e]]}{\sum A_j[(1 + \Phi_j(T)/P_e[1 + \Phi_j(T)/P_e])]}$$

This equation is solved iteratively. The result for the solar composition is shown in Fig. 9.10. The hydrogen controls the electron pressure at high temperatures. The metals dominate at cooler temperatures.

### 2.5. Complete model

The model is complete when we obtain iteratively the distribution of the temperature and gas and electron pressure. Then we can find for it the geometrical depth scale by integrating the reciprocal opacity with the optical depth. The result strongly depends on the surface gravity:

- Thickness of the atmosphere is inversely proportional to the surface gravity (Fig. 9.11).

The spectrum from the atmosphere is computed using expressions for the emergent flux:

$$F_v = 2\pi \int S_v E_\nu(\tau_v)d\tau_v$$

where the source function is assumed to be Planck function in LTE. The integrand is the flux contribution function. It is similar to that of the intensity but now include also integration over the disk (in principle limb darkening should be taken into account).
Fig. 9.10. The relation among $P_e$, $P_s$, and $T$ is shown for material having normal solar chemical composition. The dashed lines show the points corresponding to three models. The surface gravity is $10^8$ cm/s$^2$ for the two hotter models. Optical depth, $\tau_0$ in decades is denoted by the circles with the lowest ones having $\tau_0 = 10^{-3}$ and the highest ones 10. The cooler models show a nearly constant $P_e$ over a significant range in depth.

Fig. 9.11. Geometrical depth is nearly a linear function of $\log \tau_0$. The turn-down in the very shallow layers is a result of starting the integration and does not represent physical behavior. Each curve is labeled with its $S_p$, $\log g$ value. Notice the great sensitivity to gravity.
Flux contribution function

The flux contribution function tells us where the surface flux originates. A comparison of contribution function at different wavelengths is done by using a standard optical depth scale, which corresponds to one selected wavelength:

- The radiation at 8000 Å is formed higher than the radiation at 5000 Å, which reflects a smaller opacity at 5000 Å (Fig. 9.12)

- The flux above the Balmer jump (at 3660 Å) arises from significantly deeper layers than the flux below it (at 3640 Å) (Fig. 9.13).
The effect of chemical composition

The pressure structure of the atmosphere depends on the chemical composition. The effect is stronger in a cooler atmosphere (Figs. 9.19 and 9.20):

- Increase in the metal abundance (metallicity) leads to decrease in gas pressure, which is due to increase of free electrons and, thus, opacity. So, the line of sight is reduced in depth of penetration and the layer at given optical depth has smaller pressure.

**Fig. 9.19.** An increase in the metal abundance (factor shown) leads to a decrease in gas pressure at each optical depth for these cool models. This is a result of the increase in opacity caused by the increase in electron donors and the subsequent reduction in the depth of penetration of the line of sight.

**Fig. 9.20.** An increase in metal abundance (factor shown) leads to an increase in the electron pressure at most depths in these cool models. The interesting crossover in the deeper layers arises when the metals are underabundant, resulting in fewer electron donors and lower opacity allowing us to see to the deeper layers where hydrogen becomes a significant contributor of electrons.
Changes with temperature

The gas pressure decreases at a given optical depth as we consider hotter and hotter models because of the dramatic increase of the opacity in hotter models. We see less deeply in hotter atmospheres (Fig. 9.21).

![Graph showing changes in gas pressure with temperature](image1)

Fig. 9.21. The gas pressure decreases at each optical depth as the model temperature rises.

The electron pressure increases with the increase of the temperature because of increasing ionization (Fig. 9.22).

![Graph showing changes in electron pressure with temperature](image2)

Fig. 9.22. The electron pressure increases with increasing model temperature until a leveling off occurs where hydrogen takes over as the electron donor. In the domain of cooler temperatures, \( \log P_e \) varies approximately linearly with temperature. The \( \Omega \) of eq. (9.18) is shown on the left.
Model parameters

1D model atmospheres are simple but moderately realistic. Their properties can be parametrize by a few parameters:

- Effective temperature (scaling the temperature structure)
- Surface gravity (scaling the pressure structure)
- Metallicity (scaling the electron and gas pressure)

Remaining issues

- Extended atmospheres of giants and supergiants are better described by spherical geometry
- LTE is acceptable for continuum but not for strong lines which influences line blanketing effect and thus the temperature structure of the atmosphere
- In case of intense mass loss, the hydrostatic equilibrium is not valid
- Turbulence should be treated in a realistic way in deeper layers.

Reference: