2 Origin of chemical elements: Big Bang to Supernovae

2.1 Introduction

The chemical elements are the building blocks of molecules. Today we have a look at the formation and synthesis of the chemical elements. We have to distinguish between the \textit{primordial nucleosynthesis} and \textit{stellar nucleosynthesis}. The majority of today’s hydrogen and helium together with traces of Li, Be, and B were formed already in the early universe about 3 minutes “after” the Big Bang, an event referred to as primordial nucleosynthesis. All heavier elements formed later in stars.

We start with a very short summary of the current status of cosmology and the early history of the Universe. Note that we give below some additional information on cosmology, which are nice to know and round up the topic. But our main focus is the primordial nucleosynthesis. In the second part we will then concentrate on stellar nucleosynthesis.

\textbf{Figure:} Chemical abundance of the Sun. It is in broad lines representative of today’s Universe. Heavy nuclei have been built up from $^1\text{H}$ in fusion.
2.2 Today’s world picture

It is fair to say that, prior to 1990 or so, the standard Big Bang cosmology rested on just a few very simple observations about the gross properties of the Universe. The last two decades has seen an explosion in the quality of cosmological data, leading to the effective determination of the values of cosmological parameters and a real “testing” of the details of the Big Bang scenario.

The recent development in cosmology has established the following “world picture”:

Our Universe, or at least the visible part of the Universe, is practically flat, i.e. the space around us just extends to infinity in all three dimensions. On large scales greater than 100 Mpc the Universe is isotropic and homogeneous. This large-scale uniformity is observationally well established from measurements of the microwave background and forms a basic assumption in theoretical models in terms of the “Cosmological Principle”: at any epoch, the Universe appears the same to all observers, regardless of their individual locations.

The Universe is obviously not completely homogeneous, since you and I exist, and so cannot be completely isotropic as we also know from a casual glance skyward. Indeed, the sky is noticeably \textit{anisotropic} (e.g. night and day, individual points of light in the sky, the Milky Way etc., the distribution of nearby galaxies) but these can be readily explained in terms of quite local structures and inhomogeneities in the Universe. Clearly homogeneity is only an approximation which we believe is increasingly valid on larger and larger scales.

Since the Big Bang the Universe was always expanding, and, during this process cooled down. First the expansion was slowing down. But since about 5 to 10 billion years ago the expansion started to accelerate. Whether the accelerated expansion will continue forever is not known and depends on the still unknown physical reason that causes the acceleration.

![Figure](image.png)

\textbf{Figure:} Contributions to the total energy density of the Universe (from Ostriker and Steinhardt, 2003, Science 300, 1909).
The constituents of the total energy density in the Universe are shown in the figure above. The luminous matter contributes only about 0.4% to the total energy density. The remaining components are dark. Only 4.4% of the total energy density is present in form of baryonic matter (i.e. the normal matter we are familiar with made out of protons, neutrons and heavier nuclei). This means that the largest part of matter (27% of total energy density) is not only dark but also non-baryonic with unknown identity. The majority of the energy density (73%) is the so called dark energy, which is responsible for the accelerated expansion of today’s Universe. Its identity is unknown but has the property that it exerts negative pressure and, therefore, it is uniformly distributed and does not clump. Note however that theories about the identity of dark energy are still speculative. There exists at least one serious alternative theory to dark energy, the M-theory, which can also explain the accelerated expansion and even describes all four fundamental reactions in one common framework based on strings.

2.3 Important concepts and processes in the early Universe

To understand the formation of H and He during primordial nucleosynthesis we have to discuss a few basic concepts the physics in the early Universe.

Cosmological redshift and Hubble parameter

The expansion of the Universe leads to a cosmological redshift because the wavelength of photons is stretched proportionally to $R$, the scale factor of the Universe.

The cosmological redshift is parameterized as

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

The expansion of the Universe explains the observation (Hubble, 1929) that on a large scale astrophysical objects such as galaxies appear to move away from us, the more distant a galaxy the faster. Note however that it is not those galaxies that move around, rather it is space itself that expands. Therefore, the cosmological redshift is not due to the Doppler effect. In fact, the (relativistic) formula for Doppler shift differs completely from the cosmological redshift as soon as velocities approach the speed of light. The redshift-distance relation is given by

$$z = \frac{H_0}{c} d$$

with $c$ the speed of light and $d$ the distance. Observations give the following present-day (i.e. for the Universe at the present epoch) value of the Hubble constant:
More generally, the Hubble parameter is not constant, but only describes the expansion rate of the Universe, which clearly changes with time. In this sense the Hubble parameter can be given as

$$H_0 = \frac{\dot{R}}{R}$$

**Annihilation temperature for particle species**

Associated with each particle of mass $m_p$, there is a characteristic threshold temperature $T_p$ such that

$$kT_p = m_p c^2$$

The significance of $T_p$ is two-fold: Firstly, at temperatures above $T_p$ the particles will be relativistic. Similarly, above this threshold, particle/anti-particle pairs can in principle (i.e. on energetic grounds) be continually created and destroyed in reactions such as:

$$p + \bar{p} \leftrightarrow \gamma + \gamma$$

In reactions such as the one given above, equilibrium will be set up between the number densities of the relativistic particles and the photons that will represent rough equality, to within factors of order unity, in the numbers of photons and particles.

Below the threshold temperature $T_p$, there will be virtually no photons (or any other particles) with sufficient energies to create new $p\bar{p}$ pairs to replace those being annihilated. Consequently the net effect is for all the existing particles/anti-particles to annihilate. Each species would, in principle, completely annihilate as the temperature fell through $T_p$ unless there was a small net excess of particles over anti-particles, which there was. We – made as we are of matter – owe our existence to this.

**Reaction rates and freeze-out**

Many of the more interesting thermodynamic effects in an expanding Universe occur when thermodynamic equilibrium is lost. This occurs usually because of the finite reaction timescales of the reactions that are required to maintain equilibrium.

All reactions in the Universe, including the annihilation/creation reactions discussed above will have an associated reaction rate and, the inverse of this, a characteristic reaction timescale. The reaction rate can usually be expressed as the product of a velocity, a number-density and a reaction cross-section. The velocity and often the cross-section will generally depend on the temperature (i.e. energy of the particles). For
a given model of the early Universe, we know the number density etc. as a function of temperature.

The equilibrium state between reagents is usually also a function of temperature through Boltzmann terms in the energy associated with the reaction. The equilibrium state represents a balance in the reaction rates in the forward and backward directions when the number densities of reagents have changed appropriately. The reaction rate describes the rate at which equilibrium between the two sides in a given reaction is approached.

Now, an extremely important but slightly subtle point is that, if the reaction rate is slower than the rate at which the Universe is expanding, i.e. the rate at which the temperature is changing, then equilibrium will not be attained. Put another way, if the reaction timescale is longer than the age of the Universe at the epoch in question, then the reactions in question can be considered not to be occurring, even though it may be energetically favorable for them to do so.

Thus, as the Universe expands, we may find situations in which the reaction rate is falling with temperature much faster than the expansion rate (which also falls in a decelerating Universe). In this case, the reaction will start off at very early times in thermodynamic equilibrium and the ratio between the two sides of a given reaction will initially be able to track the equilibrium state as the temperature drops. However, when the two reaction rates cross and the expansion rate starts to exceed the reaction rate, the reactions effectively cease. Consequently the relative numbers of the reagents on each side of the reaction become fixed at the equilibrium state that they were in at the point where the reaction and expansion rates crossed.

This cessation of reactions leading to the reagents in a reaction becoming fixed in a non-equilibrium state is called freeze-out.
Figure: History of the Universe.
2.4 Primordial Nucleosynthesis

We start the discussion of the primordial nucleosynthesis at the time $\tau \sim 1$ sec after the Big Bang, when the Universe cooled down to a temperature of about $T \sim 10^{10}$ K. At that epoch the n/p (neutron/proton) freeze-out took place. This event and the processes over the next few minutes imprinted the characteristic chemical abundance on the Universe.

The abundances of the light elements had long been a mystery (a) due to the very uniform abundance of $^4$He and (b) to the ease with which many of the other very fragile species (e.g. $^3$H, $^7$Li) are destroyed in stars. The detailed computations of nucleosynthesis are necessarily complex, but the following simple approach gives the correct orders of magnitude and indicates the dependencies of the various abundances to variations in certain key parameters.

Before discussing these processes, it should be noted that the key difference between nucleosynthesis in the primeval fireball and that in the interiors of stars is that in the latter there is sufficient time for equilibrium to be reached amongst the different species. This is not the case for reactions in the early Universe where we are dealing with highly non-equilibrium conditions. It is the way in which equilibrium is lost in the early Universe that leaves a readily observable imprint on the processes that can be used as a powerful observational verification of our predictions for the early Universe.

The n/p freeze-out

During the era $10^{-5} \, \text{s} < \tau < 1 \, \text{s}$ protons and neutrons are able to transform into each other through the following weak interactions.

\[
p^+ + \bar{\nu}_e \Leftrightarrow n^0 + e^+
\]

\[
p^+ + e^- \Leftrightarrow n^0 + \nu_e
\]

The equilibrium ratio between these two is given as a function of Temperature by the Boltzmann factor arising from the mass difference between proton and neutron (roughly 1.3 MeV).

\[
\frac{n_n}{n_p} = \exp \left(-\frac{\Delta m c^2}{kT} \right)
\]

The weak interaction timescale for these reactions starts to exceed the expansion timescale (as calculated in the standard Friedmann models) as the temperature falls through $10^{10}$ K, corresponding to an epoch of $\tau \sim 1$ second. At this point the reactions effectively cease and the neutron fraction is frozen in at the value it had at this temperature. Substitution of $kT = 1$ MeV into the last equation gives a neutron fraction of 0.22. The reactions are further quenched when the $e^-e^+$ density drops catastrophically at $T \sim 5 \times 10^9$ K, at which point the electrons and positrons become non-relativistic and...
mutually annihilate, leading to a significant reduction in the number of potential reagents in the reactions above.

So, after \( \tau \sim 1 \) sec, the neutron fraction \( n_\infty/(n_\infty+n_p) \) is fixed at \( \sim 0.18 \) (in this simple minded approach – a more careful treatment yields a slightly different answer, see below).

If left to their own devices, the free neutrons, which are unstable to \( \beta \)-decay with a half-life of approximately 10.6 minutes unless bound to protons in stable atomic nuclei, would eventually decay into protons. Fortunately, however, nuclear reactions occur, which bind the neutrons into stable nuclei before this \( \beta \)-decay of the free neutrons has progressed very far, preserving the signature of the initial freeze-out.

### Primordial helium formation

The first process of interest is:

\[
p^+ + n^0 \Rightarrow ^2H + \gamma
\]

However, the deuteron \(^2H\) is very loosely bound with a binding energy of only 2.2 MeV and there are enough high energy photons around to photo-dissociate the \(^2H\) until the temperature drops substantially, to around \(10^9\) K. While waiting for this, the neutron fraction will continue to slowly drop due to the \(\beta\)-decay of free neutrons. Below \( T \sim 10^9 \) K, i.e. at epochs \( \tau > 100 \) sec, the photon background is sufficiently soft that \(^2H\) nuclei can survive and subsequent nuclear reactions building heavier elements out of the \(p\) and \(n\) can now take place. The free neutrons are then quickly taken up into stable nuclei, primarily \(^4He\). This hold-up in nuclear processing that occurs between 1 sec < \( \tau < 100 \) sec due to the photo-dissociation of \(^2H\) is called the deuterium bottleneck.

Once the Deuterium can survive, the following reactions will quite rapidly occur:

\[
^2H + ^2H \Rightarrow ^3He + n^0
\]

\[
^2H + ^2H \Rightarrow ^3H + p^+
\]

\[
^3H + ^2H \Rightarrow ^4He + n^0
\]

\[
^3He + ^2H \Rightarrow ^4He + p^+
\]

Unless the baryonic density is extremely low these reactions will go almost to completion, and we can assume that almost all the neutrons that have survived to the point where the \(^2H\) bottleneck is overcome, eventually end up in \(^4He\) which is the most tightly bound light atomic nucleus. Only trace amounts of the other intermediate species (\(^2H, ^3H, ^3He\)) will remain.

If we take our previous neutron fraction of 0.18, neglect the free-neutron decay, and assume that the \(^4He\) producing reactions go through to completion then the mass fraction of \(^4He\), \(Y\), is given by:
\[ Y = 1 - \left( \frac{n_p - n_n}{n_p + n_n} \right) = 2 \left( 1 + \frac{n_p}{n_n} \right)^{-1} \approx 0.36 \]

If the freeze-out and especially the neutron decay waiting for the \(^2\)H bottleneck to be overcome are computed carefully then a more precise estimate is actually slightly lower:

\[ Y \approx 0.24 \]

This is remarkably close to the observed value \( Y_p = 0.235 \pm 0.01 \).

### \(^4\)He ratio: a coincidence?

The production of roughly 25\% \(^4\)He in the early Universe is a fundamental prediction of the standard Big Bang model of the Universe, because \( Y \) is very insensitive to the total energy density (one of the main parameters in Big Bang models) and to the energy density of baryons. That this abundance is observed is thus a strong indication that we know the expansion rate and temperature of the Universe when it was 1 second old and had a temperature of \(10^{10} \) K and hence that the Big Bang really happened.

This test of the standard Friedmann Big Bang model arises because of a coincidence. The temperature at which the weak interaction rate (a function of the number density of the particles as well as weak interaction physics) drops below the expansion rate (independent of either the density of particles or of weak interaction physics) just happens to be comparable to the mass difference between the proton and neutron, the two reasonably stable hadrons out of which everyday matter is made. This was crucial for fixing the neutron number density and thus \(^4\)He. Furthermore, there is, as far as we know, no compelling anthropic argument as to why the \( Y \) abundance would have to be close to this value of 0.25 for our own existence to be possible. Certainly much lower values \( Y < 0.25 \) would be quite alright to make long-lived stars etc. On the other hand one might be able to argue that values of \( Y \sim 1 \) (a Universe composed entirely of \(^4\)He) might produce only very short lived stars making life impossible.

### The predicted abundances of the other light elements

The predictions for the abundances of the other main light elements, \(^2\)H, \(^3\)He and \(^7\)Li are more complex. These are trace components left over when most of the neutrons were taken up in the \(^4\)He. In contrast to \(^4\)He, the abundances of these minority species depend strongly on the baryon density.

\( i \) The abundances of \(^2\)H and \(^3\)He

\(^2\)H, \(^3\)H and \(^3\)He will burn to \(^4\)He for as long as the reaction timescale is less than the expansion timescale. The dependence on the baryon density (which determines the particle density in the Universe) enters because a high density Universe will:
a) overcome the Deuterium bottleneck *earlier* because the photon-particle ratio is lower.
b) burn $^2\text{H}$ and $^3\text{He}$ *faster and for longer* because the particle density is higher and therefore reaction timescales are shorter.

The net effect is that more $^2\text{H}$ and $^3\text{He}$ are transmuted into $^4\text{He}$ and their final abundances are lower for a higher baryon density. Of course, this increases the abundances but the fractional effect is very much smaller.

**Figure**: Mass fraction of nuclei as a function of temperature for $\eta=5.1\times10^{-10}$.

(ii) The abundance of $^7\text{Li}$

$^7\text{Li}$ is heavier than $^4\text{He}$ and is produced mainly by the following sequence:

$$^3\text{H} + ^4\text{He} \rightarrow ^7\text{Li}$$

$$^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be} + \gamma \rightarrow ^7\text{Li} + e^+$$
In general terms, a higher baryon density produces more $^7\text{Li}$ because there is more time for these reactions. However the dependence is complex and a very low baryon density produces a high yield of $^7\text{Li}$ because of the higher abundance of $^3\text{H}$ and $^3\text{He}$. This means that the abundance of $^7\text{Li}$ does not scale monotonically with the baryon density.

(iii) Why does nucleosynthesis stop at $^4\text{He}$: comparisons with stellar nucleosynthesis?

Apart from minute amounts of $^7\text{Li}$ and extremely small amounts of other elements, Big Bang nucleosynthesis stops at $^4\text{He}$. The simple reason for this is that there are no stable nuclei with mass numbers of 5 and 8. The most abundant species at any time are $n^0$, $p^+$ and $^4\text{He}$. Reactions combining $^4\text{He}$ with either $n^0$, $p^+$ or another $^4\text{He}$ do not produce stable nuclei. While nucleosynthesis could and does go ahead by combining $^4\text{He}$ with the rarer $^2\text{H}$, $^3\text{H}$ and $^3\text{He}$ (see above), these reactions cannot compete with the reactions making more $^4\text{He}$ out of these reagents.

There are two related differences between Big Bang nucleosynthesis and stellar nucleosynthesis that arise from the fact that, unlike in stars, the reagents are far from thermodynamic equilibrium.

Firstly, the expanding Universe is a highly non-equilibrium environment with rapidly changing density and temperature. The action of nucleosynthesis is all over in a few minutes at most. This means (a) that there is not enough time for the "slow" weak interactions that are the basis for stellar nucleosynthesis, and (b) that there is not time for statistically rare processes such as the triple-$\alpha$ process ($^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be}; ^8\text{Be} + ^4\text{He} \rightarrow ^{12}\text{C}$ before the $^8\text{Be}$ decays back to two $^4\text{He}$) to occur.

Secondly and fortunately, there is an abundance of free neutrons available to be incorporated into the heavier nuclei. This obviates the need to wait for the weak interaction to transmute protons into neutrons as occurs in stellar fusion.

The observed abundances of the light elements

The observed abundances of light elements are consistent with the theoretical predictions of the standard Big Bang theory. This is a big success and turns primordial nucleosynthesis into one of the most important tests of cosmology and Big Bang models.

The famous figure expressing this great success is shown on the following page. In contrast to the abundance of $^4\text{He}$, the abundances of the other light elements do depend strongly on the present baryonic density of the Universe. Observed abundances are indicated by the boxes. Somewhat remarkably, despite spanning a range of almost $10^9$ in relative abundance, these light element abundances are all more or less consistent with a single value of the baryon density (which is of course necessary if the theory is correct). Thus the theory of primordial nucleosynthesis can be used to determine the baryon density in the Universe.
2.5 Structure formation

All heavier elements formed later in stars. Therefore, we have a brief look at the formation of the first stars. The first galaxies and stars formed about 400 million years after the Big Bang. This requires that matter started to cluster relatively quickly from the initially almost homogeneous distribution. Experimental proof of early structure formation is found in the cosmic microwave background.
Cosmic microwave background (CMB)

About 300'000 to 400'000 years after the Big Bang the Universe has cooled to about 3'000 K, which is cool enough for electrons and protons to combine to form neutral hydrogen. This event is called recombination although it happened for the first time (the term recombination is actually a misnomer). As a consequence the interaction rate between photons and matter drops drastically, because Thomson scattering, which becomes relevant now, has a much smaller cross-section. Soon after recombination the Universe becomes practically transparent for the photons (reaction rate drops below the expansion rate) and the photons decouple from matter. These photons have a black-body spectrum and cool with the expansion of the Universe. Today they form the cosmic microwave background (CMB) with a temperature of $2.725 \pm 0.002$ K.

The observations have shown that the CMB is extremely isotropic with fluctuations in the order of $10^{-5}$, consistent with an approximately homogeneous Universe on very large scales (above 100 Mpc). This uniformity is one compelling reason to interpret the radiation as remnant heat from the Big Bang; it would be very difficult to imagine a local source of radiation that was this uniform. In fact, the CMB is one of the best experimental proofs of the Big Bang theory.

The isotropy of the CMB is explained by the so called inflation that might have taken place in the very early Universe, about $10^{-40}$ s after the Big Bang. At that time the Universe scale factor $R$ (which measures the size or typical distance scale of the Universe) might have increased enormously by as much as $10^{50}$. Such an inflation would result in the flat Universe observed today and would naturally explain the large scale homogeneity and isotropy, irrespective of the initial conditions. Furthermore, inflation would blow up the size of tiny inhomogeneities, which are present due to quantum fluctuations. These density fluctuations would later naturally form the “small-scale” structure, i.e. galaxy clusters and galaxies, and they are responsible for the tiny fluctuations observed in the CMB.
**Formation of the first stars**

The process that led to the creation of the first stars was very different from present-day star formation.

The dark matter and ordinary matter was initially mixed. According to this computer simulation, at around 100 million years \((z=24)\) after the Big Bang some dark matter concentrations began to condense into a network of filaments and sheets. Unlike ordinary matter, however, the dark matter either cannot or mostly did not collapse into dense objects like stars, brown dwarfs, and stellar remnants (white dwarfs, neutron stars, and black holes). However, the concentration of the dark matter attracted hydrogen and helium gas through gravitation.

At the nodes of the dark matter filaments, the gas clouds collapsed under gravitation towards the cores of denser clumps of \(10^5\) to \(10^6 \, M_\odot\), with \(M_\odot\) the mass of the Sun. Dark matter remained dispersed. In this way protogalaxies were formed.

Further collapse into stars began about 200 to 400 million years after the Big Bang. Cooling by molecular hydrogen could have allowed ordinary matter at the larger nodes of the dark matter filament network to fragment and collapse into individual stars of \(100\) to \(500 \, M_\odot\).

The first star-forming clumps were much warmer than the molecular gas clouds in which most stars currently form. Dust grains and molecules containing heavy elements cool the present-day clouds much more efficiently than molecular hydrogen to temperatures of only about \(10\) K. The minimum mass that a clump of gas must have to collapse under its gravity is called the Jeans mass, which scales with temperature as \(T^{3/2}\), with \(T\) the gas temperature (cf. Lecture 2). This is the reason why the first generation stars were so extremely massive. It is believed that stars of several \(100 \, M_\odot\) can only exist because they consist only of H and He. Heavier elements would strongly increase the opacity inside the star, and the radiation pressure would become so large that the star becomes unstable and is pushed apart.

What effects did these first stars have on the rest of the universe? An important property of stars with no metals is that they have higher surface temperatures than stars with compositions like that of the sun. The production of nuclear energy at the center of a star is less efficient without metals, and the star would have to be hotter and more compact to produce enough energy to counteract gravity. Because of the more compact structure, the surface layers of the star would also be hotter, possibly as hot as \(100\,000\)
K, about 17 times hotter than the Sun. Therefore, the first starlight in the universe would have been mainly ultraviolet radiation from very hot stars, and it would have begun to heat and “re-ionize” the neutral hydrogen and helium gas around these stars soon after they formed. Gradually, the first stars created ever-wider bubbles of clearer space.

The first generation of stars consisting purely of H and He, are called population III stars. Due to their large masses they had a short lifetime of a few million years. They ended their lives with a supernova explosion.

Computer simulations indicate that, as long as these very massive stars were larger than $280 \, M_\odot$ or smaller than $140 \, M_\odot$ (but more than $40 \, M_\odot$) the supernovae generated by such stars should have created black holes without significant mass ejection and so these stars would not have contributed significantly to the metal enrichment of the surrounding medium. It is possible that some of these black holes became concentrated in the inner part of large galaxies and seeded the growth of the supermassive black holes – millions of times more massive than the Sun – that are now found in galactic nuclei.

First stars born in the range of 140 to 280 $M_\odot$ ended their short lives quite differently, however. In theory, a supernova implosion from such an "intermediate" massive star would create a giant thermonuclear explosion that leaves no remnant. It is important for the chemical element composition in the universe that in the cores of population III stars the elements H and He were transmuted by nuclear fusion into heavier elements, up to Fe. These heavy elements were subsequently ejected into the interstellar medium by the terminal supernova explosion, in which even heavier elements were generated.

![Figure: Comparison of the Sun and the first stars. Computer simulations have given scientists some indication of the possible masses, sizes and other characteristics of the earliest stars. The list above compares the best estimates for the first stars with those for the Sun. An additional important difference is the composition: the first stars consisted only of H and He (with negligible amounts of heavier elements), whereas the Sun contains about 2% of heavier elements (by mass).](image)
Mixed together with fresh H and He, the enriched material then accumulated into the next generation of stars, the population II stars (“metals”, i.e. elements heavier than He, about 1/10 to 1/1000 times less abundant than in the Sun). The presence of heavy elements metals has changed the cooling properties of the gas and reduced the size of the subsequent generations of stars. By accretion into massive stars with short lifetimes, the process of enrichment of heavy elements continued over several generations of stars, until finally the metal-rich population I star chemical element mixture formed, which was the material out of which our Sun and the planets were made.

This nucleosynthesis cycle, the increase of the cosmic abundance of heavy elements over many generations of stars, was a necessary prerequisite for the formation of life. In fact, every single heavy element in our body was once produced inside a star.

**Figure**: Nucleosynthesis cycle. Interstellar material collapses to form stars, which return gas to the interstellar material after a supernova explosion or in terms of a planetary nebula. This cycle increases the cosmic abundance of He and heavier elements which are formed inside stars by fusion processes. White Dwarfs, Neutron Stars, and Black Holes contain material which is removed from the cycle.

The nucleosynthesis cycle can be interrupted by collisions between spiral galaxies. Such a collision destroys the disk of the galaxies because their systematic rotational movement is lost and because part of the interstellar gas is lost to the intergalactic material. A merger of two galaxies initially leads to a starburst because the increased pressure and density quickly forms young stars out of the interstellar gas. The interstellar material is however quickly used up. Therefore, the nucleosynthesis cycle can stop, and the galaxy ends up as a “dead” elliptical galaxy. Fortunately, or maybe not surprisingly, we live in a galaxy with a still intact accumulation of heavy elements.
2.6 Basics of stellar evolution

In this section we have a brief view at some basics of stellar evolution.

Stars and planetary systems form inside collapsing clouds of interstellar material (details of star and planet formation are discussed in Lecture 2).

Stars are only stable if the gravitational force due to their own mass, which tries to make the star collapse, is counterbalanced by a force pushing outwards. During most of the lifespan of a star this outward force is built up thanks to fusion process inside the star. The energy released in the fusion of light elements increases the temperature and pressure at the center of the star, thus creating an equilibrium stage between the gravitational force and the pressure gradient.

![Hertzsprung-Russell diagram](image)

**Figure**: Hertzsprung-Russell diagram.
Young stars produce their energy by burning H to He. Once the H in the core is used up, H-burning continues in a shell around the core, which makes the star grow to a Red Giant or Supergiant. Meanwhile the core collapses until He starts to burn. Depending on the mass of the star heavier and heavier elements can be burnt subsequently. The fusion processes occurring in stars are discussed in the following section.

The final stage of the stellar life depends on its mass. A 1 M\(_\odot\) star such as the Sun loses its outer layers, which form a planetary nebula (again a misnomer, because it has nothing to do with planets), while the core ends up as a White Dwarf. Very massive stars of many solar masses end in a supernova, and the core ends up as Neutron star or as Black Hole.

In White Dwarfs and Neutron stars the gravitational force is counterbalanced by the degeneracy pressure (due to the Pauli exclusion principle) of the electrons (White Dwarf) or the neutrons (Neutron star).

An extremely useful tool for the description of stellar evolution and for stellar classification is the Hertzsprung-Russell diagram. It shows the luminosity (total emitted energy) versus the temperature of a star. A star spends the longest phase of its lifetime (the H-burning phase) on the so called main sequence. The position on the main sequence is a unique function of the initial mass of the star: the more massive a star, the larger its radius, the larger the luminosity, the larger the temperature, and shorter its lifetime. After the main sequence phase the star moves to the upper right in the Hertzsprung-Russell diagram where all the Giants and Supergiants are located. The White Dwarfs and Neutron stars are found in the lower left of the diagram.

Figure: Spectral classes.
Stars are often classified according to their spectral class and according to their luminosity class.

The spectral classes are based on properties of the spectra. Since the line strengths depend on temperature (i.e. the spectra look different depending on the surface temperature of the star), the spectral classes can in principle replace the temperature axis in the Hertzsprung–Russell diagram (see above in the Figure of the Hertzsprung–Russell diagram). A popular mnemonic for remembering the order is "Oh Be A Fine Girl, Kiss Me", which is the sorted according to decreasing temperature (with O stars being the hottest ones. Spectral classes are further subdivided by Arabic numerals (0-9). For example, A0 denotes the hottest stars in the A class and A9 denotes the coolest ones. The Sun is classified as G2.

Stars are also classified by luminosity class. The luminosity class designation describes the size of a star from the atmospheric pressure. For larger stars of a given spectral type, the surface gravity decreases relative to what it was on the main sequence, and this decreases the equivalent widths of the absorption lines. The luminosity class is added in Roman numerals after the temperature spectral class. The full classification e.g. of the Sun is G2 V.

<table>
<thead>
<tr>
<th>Luminosity Class</th>
<th>Description</th>
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<tbody>
<tr>
<td>Ia</td>
<td>Luminous Supergiants</td>
</tr>
<tr>
<td>Ib</td>
<td>Less luminous Supergiants</td>
</tr>
<tr>
<td>II</td>
<td>Bright Giants</td>
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<tr>
<td>III</td>
<td>Giants</td>
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<tr>
<td>IV</td>
<td>Subgiants</td>
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<tr>
<td>V</td>
<td>Dwarfs (=Main Sequence Stars)</td>
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</tbody>
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2.7 Nucleosynthesis in stars

In this section we study the nuclear reactions responsible for building up heavy elements in stars. We follow the evolution of a very massive star, but many of the discussed processes such as H-burning apply also for low-mass stars. However, in a low-mass star, the formation of heavier elements stops earlier and does not go on all the way to Fe. Special emphasis is put on the evolution of population III stars.

Gravitational collapse increases temperature in the stellar core. When the temperature \( \sim 10^7 \) K is reached, the hydrogen fusion can start.

In fusion reactions light elements are transformed into heavier ones. The final products have smaller mass than the initial nuclei. The mass difference is transformed into energy:

\[
E = \Delta mc^2
\]

Fusion hydrogen into helium:

\[
m_p = 1.673 \times 10^{-24} \text{ g}, \quad m_{He} = 6.644 \times 10^{-24} \text{ g}
\]

\[
\Delta m = 4.6 \times 10^{-26} \text{ g} = 0.7\%
\]

\[
E = 4.1 \times 10^{-15} \text{ erg} = 26.7 \text{ MeV}
\]

This is called the binding energy: the energy which is needed to break the nucleus apart into its constituent protons and neutrons.

Hydrogen fusion (pp-chains):

The proton-proton (pp) chain is the only possibility for a H-He star (the same reactions take place in the Sun):

\[
\text{ppI:} \quad (1) \quad ^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e \quad \sim 10^9 \text{ years (“proton decay”)}
\]

\[
^1\text{H} + ^1\text{H} + e^- \rightarrow ^2\text{H} + \nu_e
\]

\[
(2) \quad ^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma
\]

\[
(3) \quad ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}
\]

For each reaction (3) the reactions (1) and (2) have to take place twice.

- \( ^4\text{H} \rightarrow ^4\text{He} \) produces two positrons, two neutrinos, and radiation.
- The first reaction is very slow (it explains the long lifetime of the Sun). This is the main difference to primordial nucleosynthesis, where neutrons were already...
available and could directly be fused with protons. The few minutes available for the production of He before the Universe cooled down too much were also by far too short to wait for the formation of a neutron. The stars have this time.

- The neutrino can escape freely from the star carries away some of the energy.
- The positron annihilates with an electron and produces radiation.

The reaction (3) can be substituted by other branches of the pp chain:

\[
\begin{align*}
\text{ppII:} & \quad (3) \quad ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \\
& \quad (4) \quad ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \\
& \quad (5) \quad ^7\text{Li} + ^1\text{H} \rightarrow ^4\text{He} + ^4\text{He} \\
\text{ppIII:} & \quad (3) \quad ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \\
& \quad (4) \quad ^7\text{Be} + ^1\text{H} \rightarrow ^8\text{B} + \gamma \\
& \quad (5) \quad ^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e \\
& \quad (6) \quad ^8\text{Be} \rightarrow ^4\text{He} + ^4\text{He}
\end{align*}
\]

The energy generation rate at \( T \sim 10^7 \text{ K} \):

\[ \mathcal{E}_{pp} \propto T^4 \]

- The energy output is too small to resist the gravitational collapse of a massive star. This was in particular a problem for population III stars!
- In the Sun this is however the main fusion process.

**Hydrogen fusion (CNO-cycle):**

Carbon can be used to run the more efficient hydrogen fusion CNO cycle. This is however only possible if C is indeed present. The first stars (population III) were in deep trouble here. The pp-chain was not sufficient to prevent gravitational collapse and the CNO cycle was initially not possible because of the lack of C.
CNO cycle:

1. \(^{12}\text{C} + ^{1}\text{H} \rightarrow ^{13}\text{N} + \gamma\)
2. \(^{13}\text{N} \rightarrow ^{13}\text{C} + \text{e}^+ + \nu_e\)
3. \(^{13}\text{C} + ^{1}\text{H} \rightarrow ^{14}\text{N} + \gamma\)
4. \(^{14}\text{N} + ^{1}\text{H} \rightarrow ^{15}\text{O} + \gamma\) slow reaction, \(\sim 10^4\) years
5. \(^{15}\text{O} \rightarrow ^{15}\text{N} + \text{e}^+ + \nu_e\)
6. \(^{15}\text{N} + ^{1}\text{H} \rightarrow ^{12}\text{C} + ^{4}\text{He}\)
   \(\rightarrow ^{16}\text{O} + \gamma\) probability \(\sim 10^{-3}\)
7. \(^{16}\text{O} + ^{1}\text{H} \rightarrow ^{17}\text{F} + \gamma\) slowest reaction, \(\sim 10^6\) years
8. \(^{17}\text{F} \rightarrow ^{17}\text{O} + \text{e}^+ + \nu_e\)
9. \(^{17}\text{O} + ^{1}\text{H} \rightarrow ^{14}\text{N} + ^{4}\text{He}\)

5 (p, \(\gamma\)) reactions
3 proton ‘decays’
1 (p,\(\alpha\)) reaction

T < 18\times10^6\) K pp-chain
18\times10^6\) K < T < 22\times10^6\) K CN-cycle (reactions 1 to 6)
T > 22\times10^6\) K CNO-cycle (reactions 1 to 9)

Therefore, the Sun relies mainly on the pp-chain. More massive stars burn H mainly through the CNO-cycle. The energy generation rate at T = 20\times10^6\) K:

\[
\varepsilon_{\text{CN}} \propto T^{18} \\
\varepsilon_{\text{CNO}} = 3\varepsilon_{\text{CN}}
\]

Chemical composition changes in the core:
- \(^{1}\text{H}\) reduced, \(^{4}\text{He}\) increased
- \(^{12}\text{C}\) reduced, \(^{14}\text{N}\) increased (in CNO-cycle)

Helium fusion:

If the H in the core is used up the gravitational collapse continues until the central temperature increases up to \(\sim 10^8\) K. Once it is achieved, helium can fuse into carbon – a new energy source. The higher temperature is needed, because the electric charge of a helium nucleus is twice that of a hydrogen nucleus and they need higher velocities (and hence higher temperatures) to overcome the electrostatic repulsion.
Triple alpha process: $3 \, ^4\text{He} \rightarrow ^{12}\text{C}$

The fusion process which combines three helium nuclei, or alpha particles, into one carbon nucleus is called the triple alpha process.

1. $^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be}$
2. $^8\text{Be} + ^4\text{He} \rightarrow ^{12}\text{C} + \gamma$ (sometimes)
3. $^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O} + \gamma$

$^8\text{Be}$ is unstable and decays into two $^4\text{He}$ particles in $2.6 \times 10^{-16}$ s, so that the three $^4\text{He}$ particles should collide almost simultaneously. Fortunately, $^{12}\text{C}$ has an excited state at 7.56 MeV, which is very near to the 7.4 MeV of the emitted photon. Therefore, this $^{12}\text{C}$ resonance greatly increases the probability of the triple alpha process. The astrophysicist Sir Fred Hoyle even predicted the existence of this $^{12}\text{C}$ resonance based on the anthropic principle and the “necessity” for carbon to be produced in stars.

The energy generation rate at $T \sim 10^8$ K:

\[ \varepsilon_{3\alpha} \propto T^{40} \]

- The energy output is large enough to resist the gravitational collapse, even in the case of population III stars. This saves the population III stars: the He formed by the pp-chain is burnt into C (thus stopping the collapse); once enough C is produced it can also rely on the more efficient CNO-cycle for the He formation.
- Carbon is produced in the stellar core.

**Carbon fusion:**

Helium fusion produces a carbon-oxygen core.

C-O Core collapses until:

- $T_c > 600 \times 10^6$ K
- density $> 1.5 \times 10^{5}$ g cm$^{-3}$
- ignites carbon burning in the core

\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg} + \gamma \]
\[ \rightarrow ^{23}\text{Na} + ^1\text{H} \]
\[ \rightarrow ^{20}\text{Ne} + ^4\text{He} \]
\[ \rightarrow ^{23}\text{Mg} + n \]
\[ \rightarrow ^{16}\text{O} + 2 \, ^4\text{He} \]
Neon fusion

O-Ne-Mg core contracts & heats up until:
- $T_{\text{core}} \sim 1.5 \times 10^9$ K
- density $\sim 10^7$ g cm$^{-3}$

Ignite Neon burning:
- reaction network makes O, Mg, & others
- Huge neutrino losses: $> L^*$!
- Builds a heavy O-Mg core
- Lasts for a few years before Ne runs out

Oxygen fusion

Ne runs out, core contracts & heats up until:
- $T_{\text{core}} \sim 2.1 \times 10^9$ K
- density $\sim$ few $\times 10^7$ g cm$^{-3}$

Ignite Oxygen burning:
- reaction network making Si, S, P, Mg
- Huge neutrino losses: $> 100,000$ L$^*$!
- Builds a heavy Si core
- Lasts for $\sim$ 1 year before O runs out.

$$^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S} + \gamma$$
$$\rightarrow ^{31}\text{P} + ^{1}\text{H}$$
$$\rightarrow ^{28}\text{Si} + ^{4}\text{He}$$
$$\rightarrow ^{31}\text{S} + n$$
$$\rightarrow ^{24}\text{Mg} + 2 \ ^{4}\text{He}$$

Silicon fusion

O runs out, Si core contracts & heats up until:
- $T_{\text{core}} \sim 3.510^9$ K
- density $\sim 10^8$ g cm$^{-3}$

Ignite Silicon burning:
- Si melts into a sea of He, p, & n
- Fuses with rest into Nickel (Ni) & Iron (Fe)
- Builds a heavy Ni/Fe core
- Lasts for about 1 day...
The core of a massive star at the end of Silicon burning is made up of many shells, resembling the different layers of an onion:

Iron sets an important endpoint to the fusion of ever heavier elements. The iron nucleus (containing 26 protons) is the most tightly bound of all nuclei. The fusion of iron nuclei absorbs energy, instead of emitting energy.
At the end of the Silicon Burning Day:
- Build up an inert Fe core
- Onion Skin of nested nuclear burning shells

Finally, the Fe core exceeds $1.2-2 \, M_\odot$:
- Fe core begins to contract & heat up.
- Collapse resulting in a supernova explosion

The exact amount of metals that are blown out by supernovae into the surrounding ISM is unknown. It depends on the exact position of the radius at which all the above material is ejected. The position of the mass-cut also determines which metals are ejected.

![Crab nebulae, SN 1054](image)

**Synthesis of heavy elements**

The buildup of heavier elements in the nuclear fusion processes in stars is limited to elements below iron, since the fusion of iron would subtract energy rather than provide it. $^{56}\text{Fe}$ is abundant in stellar processes, and with a binding energy per nucleon of 8.8 MeV, it is the third most tightly bound of the nuclides. Its average binding energy per nucleon is exceeded only by $^{58}\text{Fe}$ and $^{62}\text{Ni}$, the nickel isotope being the most tightly bound of the nuclides.

The peak in the binding energy curve near iron means that more tightly bound nuclei are formed either by the breakup of heavier nuclei (fission) or neutron capture:

$$^{\text{A}}\text{E} + n \rightarrow ^{\text{A+1}}\text{E} + \gamma$$

$$^{\text{A+1}}\text{E} \rightarrow ^{\text{A+1}}\text{D} + e^{-} \quad \text{(beta-decay)}$$

**s-process**
- slow neutron capture
- low neutron fluxes ($10^5$ to $10^{11}$ neutrons cm$^{-2}$ s$^{-1}$)
- late stages of stellar evolution (during C, O, Ne and Si burning)

**r-process**
- rapid neutron capture
- high neutron fluxes ($\approx 10^{22}$ neutrons cm$^{-2}$ s$^{-1}$)
- during supernova explosions
2.8 Summary

- Hydrogen (76% by mass) and Helium (24% by mass) were formed a few minutes after the Big Bang during the primordial nucleosynthesis.
- Heavier elements formed in stars and were accumulated in the nucleosynthesis cycle. Elements heavier than Fe are produced only in supernovae.
- Primordial nucleosynthesis:
  - proton to neutron ratio fixed after 1 second (freeze-out of weak interaction)
  - neutrons decay until deuterium bottleneck is overcome
  - remaining neutrons quickly used up for helium.
- Stellar nucleosynthesis:
  - H-burning slow, because no free neutrons available (⇒ stability of main sequence stars)
  - First stars (population III):
    - Very massive: 100 – 500 M⊙
    - Energy production by pp-chain not sufficient
    - Initially no CNO cycle (only H and He available!)
    - Triple alpha process stopped collapse
  - Heavy elements built up in “onion-shells” structure
  - Elements heavier than Fe only produced during Supernovae
  - Nucleosynthesis cycle: elements returned to interstellar matter by Supernovae and Planetary nebulae

This means that every atom heavier than He on Earth or in our body was once formed in a star. Every Fe atom in our blood experienced a Supernova. We are indeed children of the stars.