Spectral Analysis of Three-Dimensional Magnetohydrodynamic Models of the Solar Corona

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The cover picture provides a view into the simulation box of the solar atmosphere. The vertical magnetic field shown at the bottom of the box is scaled from -1000 G (black, inward directed field) to +1000 G (white, outward directed field). The left and the right side of the box represent cuts of the particle number density (red: $10^{14}$ m$^{-3}$, yellow: $10^{15.5}$ m$^{-3}$, green: $10^{17}$ m$^{-3}$, blue: $10^{21}$ m$^{-3}$, violet: $10^{23}$ m$^{-3}$). The dark loop-like structures at medium heights in the simulation box illustrate the O vi (1032 Å) emission that has been calculated assuming ionization equilibrium. At the backside of the box the vertical velocity component - scaled from -5 km/s (blue, pointing upwards) to +15 km/s (red, pointing downwards) is shown. For details see section 3.2.
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Acknowledgment
Cool stars like the Sun are surrounded by a hot outer atmosphere, the corona. The corona of the Sun is a hundred times hotter than the solar surface. The cause of coronal heating is one of the fundamental problems in solar physics today. Numerous mechanisms of coronal heating have been proposed, e.g. acoustic wave heating or magnetic reconnection and dissipation processes giving rise to tiny energy release events - so-called nanoflares. In this thesis detailed analyses of three-dimensional magnetohydrodynamic (MHD) models of the outer solar atmosphere are presented. The heating mechanism in these models is based on the braiding of magnetic field lines by random plasma motions on the solar surface (Parker, 1983). The subsequent induction of currents and their dissipation results in a multitude of small-scale heating events that build up the corona. In contrast to previous models, the complex interactions of both strong and weak magnetic fields are taken into account in the MHD calculations, thus allowing an accurate characterization of the three-dimensional structure and dynamics of the solar transition region and corona.

Synthetic spectra of optically thin transition region and coronal emission lines in the extreme ultraviolet range are calculated from the simulation data. Average properties of these spectra, such as mean intensities and Doppler shifts are found to be comparable to those of observed spectra. The synthetic intensity images show both low-lying as well as high-reaching loop-like structures with similarities to structures observed with the TRACE satellite. Variations on the time-scale of minutes are being observed in both the synthetic intensity maps and the synthetic Doppler shift maps of transition region lines that reveal the reaction of the upper solar atmosphere to the photospheric plasma motions. The Doppler shifts synthesized from the model output show the expected redshifts in the transition region in accordance to spectra recorded by instruments like SUMER/SOHO. The differential emission measure function calculated from the model data shows the well-known increase for low temperatures. These comparisons provide independent results that confirm the validity of the model presented.

The investigation of coronal mass flows in the model shows that downflows are prevailing. The correlation found between line shift and line intensity indicates an overestimation of the mass downflows if these are calculated on the basis of redshifts observed in the model.
Abstract

The magnetic field topology allows the in detail investigation of the magnetic field structure surrounding a small X-type reconnection point. A new concept of a chromosphere-corona mass cycle is developed based on the observed velocities along the magnetic field lines: Magnetic field lines are intermittently connected to sites of increased heating where plasma is being injected into the coronal loop. These plasma upflows lead to the observed blueshifts in the loop. After the restructuring of the magnetic field, the field lines are connected to sites of less heating and the material flows back down along the field lines. This draining of the loop is observed as a redshift.

Data of a one hour numerical simulation run of the solar atmosphere above a small active region is subject of the last part of this work. Other than in the 30 minute simulation run, an eruption in the form of a plasma blob occurs in the middle of the time-series. Prior to the rise of the plasma eruption the formation of a soliton-like structure is observed in the simulation box. Enhanced heating causes a strong temperature increase in the formation region of the structure, and eventually leads to the onset of the eruption. Subsequently, the structure expands nearly adiabatically, with a slightly stronger temperature decrease than expected for adiabatic expansion. During the 11 minute flight time the mass of the plasma blob increases by more than a factor of two. It is shown that this mass increase is due to a continuous mass inflow into the tube that the structure passes. The plasma blob rises parallel to the magnetic field lines, and while expanding changes its shape from tailed to ellipsoidal. As it descends, the plasma blob deforms the magnetic field dynamically. As a consequence, it is slowing down being heated at its front. The comparison of the flight velocity versus the Alfvén velocity indicates that the plasma eruption is a purely hydrodynamic rather than a magnetohydrodynamic phenomenon.
Zusammenfassung


Zusammenfassung


1 Introduction

1.1 The Sun’s Outer Atmosphere

The Sun is a main-sequence star of spectral class G2 with a mass $M_{\odot} = 2 \cdot 10^{30}$ kg, a radius $R_{\odot} = 7 \cdot 10^8$ m and an effective temperature of $T_{\text{eff}} = 5780$ Kelvin (K). Due to its proximity to the Earth the Sun is the only star that can be spatially resolved by direct means. However, only the lowest layers of the Sun’s atmosphere, the photosphere and the chromosphere, can be regularly observed from the ground. The upper atmospheric layers, i.e. the transition region, the corona and the solar wind are best studied from space. The choice of suitable spectral lines allows the determination of the physical characteristics of all the layers that constitute the solar atmosphere. In the following the various atmospheric layers are discussed, starting with the photosphere and moving outward.

1.1.1 The Photosphere

The visible surface of the Sun called photosphere (Greek “light sphere” ) is the layer emitting the bulk of electromagnetic radiation. When moving outwards through this layer of a few hundred kilometers thickness, the gas changes from being completely opaque to almost completely transparent. The photosphere constitutes the coldest region of the Sun ($T \approx 4300 - 6000$ K) and the part of the solar atmosphere with the highest density (particle density $N \approx 10^{23}$ m$^{-3}$, Fig. [1.3]). The temperature minimum of $T \approx 4300$ K is reached at about 500 km above the limb at the top of the photosphere. The photosphere shows a regular pattern of convection cells, called granules, with diameters of the order of 1000 km that change on the time scale of minutes. Hot gas is rising from the interior to an altitude where the opacity becomes small enough so that radiation can escape. As the gas radiates, it cools down and descends. These motions represent a huge reservoir of mechanical energy. The perpetual motion of the Sun’s surface is evident in another level of patterning, called supergranulation. Supergranules are cells of $\approx 30,000$ km diameter and life times of about one day that are named after their resemblance to the smaller granules. Sunspots, dark regions of lower temperature ($T=4000-5000$ K) are the most obvious form of structure on the solar disk (Fig. [1.1]). They were discovered in the early 17th century following the introduction of the telescope. Almost 300 years later in 1908, G. E. Hale proved strong magnetic fields of several thousand Gauss.
in the environment of sunspots by means of the Zeeman effect. In the dark umbra the magnetic field lines are directed perpendicular to the surface, in the surrounding penumbra the angle of emergence is decreasing. The strong magnetic fields in sunspots are reducing the convection and thus the energy transport and as a consequence a decrease in temperature of approximately 2000 K is observed within spots. Sunspots range from about 1500 km to around 50,000 km in diameter. Their distribution varies periodically in an 11-year sunspot cycle. Early in the cycle they appear at high latitudes and then gradually approach the equator. Sunspots are surrounded by smaller flux concentrations of weaker field called active or bipolar regions. They emerge together with the spots and the neighboring opposite-polarity field. The photosphere reveals a wealth of small-scale magnetic structures, such as bright points, micro-pores and pores that influence and modify the photospheric emission and are subject to granular flows.

Figure 1.1: Sunspot AR 10810 observed Sept. 23, 2005 with the Dunn Solar Telescope. Speckle image reconstruction has been applied to correct for the blurring by the Earth’s turbulent atmosphere. The high-resolution G-band (430.5 nm) image shows the dark sunspot umbra surrounded by the penumbra (fluted structures radiating outward from the spot) embedded in the granulation pattern. The size of the image corresponds to 3.2 times the size of the Earth. Image credit: F. Woeger (KIS), C. Burst, M. Komsa (NSO/AURA/NSF).

1.1.2 The Chromosphere

The chromosphere (Greek "colored sphere") extends above the photosphere and is named after its colorful appearance in the form of a reddish flash shortly before and after a total solar eclipse. This color originates from the red hydrogen Balmer-α line
at $\lambda = 656.3$ nm appearing in emission above the solar limb. The chromosphere is much thicker than the photosphere. It contains roughly 10-12 scale heights, each of some 200 km with temperatures varying between 4000 and 25,000 K. Compared to the visible solar surface, the chromosphere is much more structured by the magnetic field. The most prominent features of the solar chromosphere are bright structures that form the chromospheric network and outline the boundaries of the supergranular flow cells. The magnetic field in the lower chromosphere is concentrated in this network. This “inverse granulation” pattern is caused by the hotter gas granules that while they are rising are expanding and cooling. The cool gas descends towards the photosphere where it converges with the flow from other expanded granules. In contrast, the network emission seems to be caused by an enhanced heating caused by the magnetic field, the nature of which is still not known.

Special narrow-band optical filters tuned to a certain spectral line, e.g. the H-$\alpha$ (6563 Å) line, allow to observe the chromosphere against the bright photosphere. The most common feature in the solar chromosphere are spicules, dynamic jets that are visible as dark tubes on the solar disk and act as agents in the transport of energy and mass in the lower solar atmosphere. Results from the Hinode satellite have revealed the existence of at least two types of spicules. “Type-I” spicules are long lived (3-5 min) and show longitudinal motions of $\approx 20$ km/s with little contribution in heating chromospheric plasma to transition region temperatures. The recently discovered “Type-II” spicules show much shorter lifetimes (50-100 s) and higher velocities (50-150 km/s). They have been found to exhibit mostly upward motion and are therefore discussed as a hot mass transport mechanism for the outer solar atmosphere (e.g. McIntosh and De Pontieu, 2009).

Both filaments and prominences characterize large loop-like features anchored in the photosphere extending outwards into the corona where the cool plasma ($T \approx 10^4$ K) is kept by inclined magnetic fields. Solar filaments are seen in absorption against the solar disk where they appear as thin threads. When observed in emission beyond the solar limb these features are referred to as solar prominences (Fig. 1.2). Solar prominences appear as bright dense clouds of material against the dark outer space ranging in size up to thousands of kilometers. Both filaments and prominences can remain in a quiescent state for days and weeks. However, as the magnetic loops that support them slowly change their structure, filaments can erupt and rise off the Sun over the course of a few minutes or hours.

1.1.3 The Transition Region

The transition region located between the chromosphere and the corona separates two very different temperature regimes. It is very thin, below 0.01% of the Sun’s diameter, but highly structured and extremely dynamic. Moving outwards in the transition region, the temperature rises rapidly from 25,000 K to $10^6$ K (see Fig. 1.3). Numerous spectral lines from a variety of ions are formed in the transition
1 Introduction

Figure 1.2: Solar prominence eruption on September 29, 2008 observed by STEREO in the 304 Å wavelength band. Image credit: STEREO Project, NASA.

region (Fig. 1.3) and constitute a powerful tool to study this layer. Virtually all of these emission lines are located in the ultraviolet (UV) and extreme ultraviolet (EUV) parts of the spectrum, wavelength regimes that are blocked by the Earth's atmosphere. Therefore, observations of this layer must be carried out from space.

1.1.3.1 Energy Balance in the Transition Region

Plasma with a temperature and density typical of the lower transition region can radiate its energy in a so-called radiative cooling time of only a few seconds, while plasma with the properties of the inner corona needs about an hour to cool by radiation. Thus, the transition region is able to rapidly respond to changes in the energy balance, and it is therefore well suited to study coronal heating processes. Major difficulties in understanding the observations are due to the complicated geometry and time-variability of the transition region. Fluctuations in the coronal heating rate generate flows through the transition region by processes named “chromospheric evaporation” and “coronal condensation”. If the energy input into the corona is temporarily enhanced, the excess energy can no longer be radiated away in the corona or the transition region. Therefore, the excess energy is transported down into the chromosphere by thermal conduction, where it heats up the plasma which then expands into the corona (“chromospheric evaporation”). On the contrary, when the heat input into the corona is temporarily reduced, the coronal plasma cools and flows back down into the chromosphere (“coronal condensation”).
1.1 The Sun’s Outer Atmosphere

Figure 1.3: Average temperature (red) and density (blue) structure of the quiet solar chromosphere, transition region and corona according to Vernazza et al. (1981). The height scale starts where the optical depth at 5000 Å is unity. The green dots indicate the formation temperature and height of prominent extreme ultraviolet emission lines that will be synthesized in this study. Courtesy of Hardi Peter.

1.1.3.2 Transition Region Dynamics

The transition region is not only complicated because of its structure, but also due to various types of motions and a high level of temporal variations. Imaging instruments like TRACE have observed variations and motions on all spatial and temporal scales - including brightenings, oscillating loops, magnetic field reconfigurations associated with mass ejections, or up- and downflowing cool gas in spicules and active-region filaments. From such observations it is often not possible to distinguish between apparent motions, such as moving wave fronts and gas motions.

A more direct measurement of plasma velocities is based on the analysis of spectral line profiles obtained by spectrometers like e.g. SUMER/SOHO. Such measurements identify motions both towards and away from the observer. The gas that contributes most to the emission from lower transition region temperatures is predominantly moving towards the Sun (visible as a redshift) at speeds reaching a maximum of 10 km/s for temperatures near $10^5$ K. The average downflow speed decreases both towards lower and higher temperatures and turns into an upward speed for temperatures higher than $10^5$ K (Fig. 1.4). Both blueshift and redshift decrease toward the limb of the Sun which indicates that the motions are primarily vertical.
The origin of line shifts in the solar transition region has been one of the long standing questions in solar physics. Systematic redshifts of transition region spectral lines are not only observed on the Sun, but also in spectra of late type stars (Ayres et al., 1988), indicating that the phenomenon causing the redshifts is of general nature. An early idea to explain this phenomenon was that plasma descending in the solar atmosphere would generate more emission than ascending plasma resulting in redshifts at all temperatures (Doschek et al., 1976). Another theory was that the observed redshifts are due to cooling and downflowing plasma after this was ejected into the corona by spicules (Pneumann and Kopp, 1978; Athay, 1984). Another class of models used to explain the redshifts involves siphon flows through cool (below $10^6$ K) coronal loops (McClymont and Craig, 1987). Stationary flows driven by asymmetric heating of the loop are causing evaporation in the hot leg of the loop and thus a flow to the cooler side. As the condensing leg with the downflow dominates the emission, a net redshift would be observed. When studying the center-to-limb variation of emission line Doppler shifts, Peter and Judge (1999) found that lines formed in the upper transition region and corona show blueshifts at disk center rather than redshifts (Fig. 1.4). Their discussion showed that models using, e.g.
flows through loops or the return of spicular material cannot deduce the observed line shifts over the whole temperature range. Hansteen (1993) proposed that the redshifts observed in transition region spectral lines are caused by downward propagating acoustic waves that are assumed to be generated in the corona as a result of small episodic heating events called nanoflares. With nanoflares occurring predominantly at the top of the loop Hansteen found that downward-propagating waves caused by the release of energy in small volumes can account for the redshift in transition region lines, although no net mass flows are present. In a more recent study, Hansteen et al. (1997) included the effect of downward propagating MHD waves being reflected in the chromosphere and found net redshifts in transition region lines and net blueshifts in coronal lines within a reasonable range. In addition, the variation of the emission line Doppler shifts deduced from these models agrees well with the observed center-to-limb variation (Peter and Judge, 1999) making these models seem a promising way to understand the transition region line shifts.

1.1.4 The Solar Corona

The corona (from Latin "crown") is the Sun’s hot outer atmosphere that extends outwards into space for many solar radii. On the Earth, it can be observed during total solar eclipses when it appears as a white crown surrounding the Sun. Fig. 1.5 shows images of the corona during a total solar eclipse near minimum (left) and maximum of solar activity (right). The emitting plasma is structured by the magnetic field lines that are mostly radial and carry energetic particles outwards to form the solar wind.

Figure 1.5: Left: Solar corona during total eclipse in Mongolia, on August 1, 2008 (near solar minimum). Right: Solar corona during total eclipse in Zambia, June 21, 2001 (near solar maximum). From Miroslav Druckmüller.
By comparing spectra from solar eclipses to laboratory data from highly ionized elements, Grotian (1939) and Edlén (1943) were the first to establish that the coronal emission arises from highly ionized elements at temperatures of more than a million degrees Kelvin. This observation - being in conflict with the basic principles of thermodynamics at first sight - has been one of the longest known problems in astrophysics. As a hot corona cannot be in static equilibrium with the interstellar medium, Parker (1958) gave the first theoretical description of a continuous outflow named solar/stellar wind. Such winds can be driven by many different processes including pressure gradients, sound waves, Alfvén waves or radiation. Later on, studies revealed that the energy loss by inward heat conduction back to the Sun is not negligible (Hammer, 1982a, 1982b; Withbroe, 1988) in comparison to the energy loss by the solar wind. This requires the inclusion of the transition region between the cool chromosphere and the hot corona where the steep temperature gradient leads to a high thermal conduction. The first models treating the complete chromosphere-to-solar wind system were developed by Hansteen and Leer (1995).

Electron densities in the solar corona are two orders of magnitude lower than in the transition region. Since for the outer solar atmosphere, the total emitted energy per volume and time is a function of the square of density times a function of temperature that peaks around $10^5$ K, the corona radiates less efficiently than the transition region.

1.1.4.1 Structure of the Solar Corona

The solar corona is closely related to the Sun’s magnetism. This is evident when observing the corona’s variation in shape during transition from minimum to maximum activity. A typical minimum corona is elongated along the equator and shows so-called polar plumes, thin hair-like structures at the poles that resemble the lines of force of a bar magnet. The maximum corona (Fig. 1.5, right) is more structured than the minimum corona (Fig. 1.5, left) and the visible features are distributed evenly over all latitudes thus forming a more circular shape. The solar magnetic field plays a dominant role for the structuring of the plasma in the lower solar atmosphere. Regions of concentrated magnetic flux, so-called flux tubes, rise through the convection zone and penetrate into the photosphere causing the formation of active regions and sunspots. In the low layers of the solar atmosphere, the gas pressure is high compared to the magnetic pressure, so that flux tubes are advected by the surrounding plasma flow. The pressure decreases exponentially with height in the solar atmosphere, whereas the magnetic field strength follows a power law and thus decreases more slowly with height compared to the pressure term. Therefore, the magnetic energy dominates the thermal energy in the transition region and corona.
The magnetic field fans out with height and eventually fills the whole corona. The high temperature of the corona results in a large number of free electrons that give rise to both a high electric conductivity and a high thermal conductivity. If the electric conductivity is high, the magnetic field is “frozen” in the plasma. As thermal conduction parallel to the magnetic field is much more efficient than thermal conduction perpendicular to the magnetic field, the plasma in the upper transition region and corona traces the structure of the magnetic field (Fig. 1.6). Most field lines close back on the solar surface and connect photospheric regions of opposite polarity. If these bundles of magnetic field lines are filled with plasma, they become visible through the emission of radiation, and are referred to as coronal loops. Coronal loops are observed to be structured down to the smallest spatial scales that can be resolved with current space-based instruments. They are likely to be composed of even smaller scale structures that are often referred to as strands. The corona is filled with loops of different sizes, ranging from small inter-netwok loops of several 1000 kilometers of length to large trans-equatorial loops that can reach lengths of several 100,000 kilometers.

**Figure 1.6:** Arcade of magnetic loops in the solar corona, as seen with the TRACE solar spacecraft in the lines Fe\textsc{ix} and Fe\textsc{x} at 171 Å. The image shows plasma at temperature $10^6$ K captured by the magnetic field. Image credit: TRACE operation team, Lockheed Martin.

### 1.1.4.2 Coronal Heating

The problem of coronal heating has puzzled solar physicists since the 1930s, when Grotian (1939) and Edlén (1943) discovered that the corona has a temperature of the order of one million Kelvin, three orders of magnitude higher than the photosphere.
at 6000 K. Usually this problem is approached by assuming the deposition of non-thermal energy at coronal heights generated by the interaction of convection and magnetic field. It is unclear though how and where coronal heating occurs and in which way the corona is filled with hot plasma. It is obvious that a kind of mechanical energy input is necessary to raise the coronal temperature higher than the photospheric one. The chromosphere is generating more than enough mechanical energy flux to heat the corona, but the question remains how it is transported to the corona and how it is dissipated. One of the first heating mechanisms discussed for the hot corona is based on upward propagating sound waves generated at or below the solar surface (Schwarzschild, 1948). Later it became evident that the heating mechanism is related to the magnetic field that is anchored in the photosphere. At the solar surface, the footpoints of the field lines are shuffled around by the photospheric motions, so that the field lines are continuously wrapped and rotated about each other (Fig. 1.7). Thus, magnetic field gradients are built up and currents are induced, the dissipation of which releases enough energy to heat the corona (Parker, 1972, 1983).

Figure 1.7: Schematic drawing of flux tube braiding as a consequence of photospheric footpoint motions leading to heating of the corona. Adapted from Parker (1983).

However, only recently the feasibility of a description of the flux-braiding mechanism in a three-dimensional (3D) magnetohydrodynamic (MHD) model (Hendrix et al., 1996; Galsgaard and Nordlund, 1996) has been shown. Gudiksen and Nordlund (2002) constructed the first 3D MHD box models of the solar corona, where the heating of the plasma in the numerical description is due to Ohmic dissipation, which
1.1 The Sun’s Outer Atmosphere

is a source term in the energy equation. In these models synthesized photospheric motions are applied that match the observed velocity and vorticity spectra of the solar photosphere. However, models put forth so far lack a major feature of the solar atmosphere, i.e. the chromospheric network. The first models to account for the complex interaction of regions of strong and weak magnetic field were developed by Bingert (2009). These models allow for a thorough investigation of the three-dimensional structure and dynamics of the transition region and low corona. Time-series of data from such models will be analyzed in this study.

1.1.5 Space-Based Extreme Ultraviolet Observations of the Sun’s Outer Atmosphere

Absorption of radiation in the Earth’s atmosphere, mostly due to O$_2$, prevents Earth-bound observations in the ultraviolet (UV) and extreme ultraviolet (EUV) range of the electromagnetic spectrum and requires rocket or satellite observations from space. In 1946, the first UV spectrum of the Sun was obtained using a captured V2 rocket. In the early 1960s whole-disk rocket spectra became available and the quantitative analysis of EUV spectra started. The EUV part of the spectrum ranges from 100 Å to 1200 Å and the UV portion of the spectrum extends from 1200 Å to 2000 Å. Below 1500 Å, emission lines dominate the spectrum. The Lyman $\alpha$ line of hydrogen centered at 1215.7 Å is the most prominent one. The strong resonance lines of ions abundant at transition region temperatures, such as H, He, C, N, O, Ne, Mg and Si in different ionization stages, are located in the EUV and UV part of the spectrum. The High Resolution Telescope and Spectrograph (HRTS) flown on the Skylab mission (1973-1974) and other rocket flights in the 1970ies and 80ies partly revealed the structure of the transition region and inspired a variety of models (Mariska, 1992). Since the Yohkoh era in the 1990s the dynamical structure of the solar corona at various time-scales has become increasingly recognized, and explosive phenomena such as flares were discovered to be associated with the changing structure of the magnetic field. More recent observations were obtained by the Solar and Heliospheric Observatory (SOHO) that was launched in 1995. The UV telescope and spectrometer SUMER (Solar Ultraviolet Measurements of Emitted Radiation; Wilhelm et al., 1995) on board SOHO was designed to perform detailed spectroscopic plasma diagnostics of the solar transition region. The instrument provides the possibility to acquire large raster scans with spectral resolution and sufficient signal-to-noise ratio thus allowing to perform reliable Gaussian fits of the spectral profiles. The Transition Region and Coronal Explorer (TRACE; Handy et al., 1999) was launched in 1998 to allow joint observation with SOHO. It produces high spatial and temporal resolution images, while SOHO yields images and spectral data up to 30 solar radii at much lower spatial and temporal resolution. TRACE allows to observe a corona that is extremely dynamic with a multitude of flows and wave phenomena. Fig. 1.6 provides an impression of the very thin fibril-like loops...
that are observed with TRACE. More recently, in 2006, the Hinode spacecraft has been launched in order to investigate the interaction between the Sun’s magnetic field and the corona. Hinode’s EUV imaging spectrometer EIS ([Culhane et al. 2007]) allows fast spatial scans in a number of hot iron lines, e.g. Fe\textsubscript{x} (185 Å), Fe\textsubscript{xii} (195 Å), Fe\textsubscript{xiii} (204 Å) and Fe\textsubscript{xv} (284 Å) formed at temperatures between one and two million Kelvin. With its high spectral resolution EIS allows to follow the dynamics of the corona in detail.

1.2 Objectives and Goals

This work aims at providing an insight into the structure and dynamics of solar-like coronae by means of spectral analysis of extreme ultraviolet emission line spectra that are calculated from three-dimensional MHD models. MHD models examined are the first to account for the complex interaction between regions of strong and weak magnetic fields by including the chromospheric network structure in the simulations. Therefore, for the first time a realistic model of the solar corona will be tested for its ability of reproducing detailed structures. The model will, at the same time, be tested for its ability of reproducing the large spatial and temporal variability of averaged key parameters.

The focus will be on the magnetically-closed corona with field lines rooted in the solar surface at both ends. Aspects of the solar wind (magnetically-open corona) will not be addressed. The heating mechanism of these models is based on the braiding of magnetic field lines that originates from photospheric plasma motions.

An emission line synthesis will be carried out for a set of emission lines that have been observed with spectrometers like SUMER/SOHO and EIS/Hinode. The line formation temperatures range from $4 \times 10^4$ to $2 \times 10^6$ K and therefore provide a continuous coverage of the transition region and corona. The characteristics of the simulated spectra will be compared with the characteristics of the observed solar spectra in order to evaluate the quality of the model. It is of interest to also investigate the variability of the spectra, a response of the upper atmosphere to the heating mechanism, in order to provide evidence that braiding of the magnetic field lines is the dominant heating mechanism in the solar corona.

If in the above considerations the model applied turns out to be reasonable it may be possible to derive some contributions to the coronal structure above the chromospheric network, interactions of strong and weak magnetic fields and causes of the flows leading to the observed redshifts and blueshifts of extreme ultraviolet emission lines.
1.2 Objectives and Goals

In chapter 2, the model and the setup of the simulation are summarized. The MHD equations governing the transition region and corona are introduced. An overview of the physical processes is provided that occur in the solar transition region and corona leading to emission in the extreme ultraviolet range of the spectrum. Furthermore, the synthesis of optically-thin emission lines from the model data will be shown as will be the calculation of spectral maps. In chapter 3 and 4, these procedures are applied to the simulation data, and results of the spectral analysis for two simulation runs are presented. In chapter 5, results will be discussed and conclusions provided.
2 Theory and Methodology

2.1 MHD Simulations of the Corona in 3D with the Pencil Code

Only since the early 2000s computer power allows numerical experiments that include the entire domain from the photosphere to the corona. One of the reasons why this option has not been available earlier is related to the large domains that need to be modeled with high resolution. Domain sizes with edge lengths of several tens of megameters are needed to involve high-reaching loops that connect active regions. For numerical reasons, the temperature gradient cannot be steeper than the grid step size (varying between 70 and 280 km). Therefore, scales of a few tens of kilometers are required to resolve the large temperature gradients in the transition region, and computational domains of 150^3 grid points or more are necessary. Even for today’s systems, this is not a trivial task. The thermal conduction term constitutes another issue that significantly limits the timestep at which codes can be stable run, as a result of its scaling with the grid size squared. Gudiksen and Nordlund (2002) were the first to overcome the numerical challenges and to model an adequate photosphere-to-corona system. Synthetic images and diagnostics of transition region lines from their models show close similarity with observed EUV images from TRACE and spectral maps from SUMER (Peter et al., 2006).

2.1.1 The Pencil Code

The solar corona above the chromospheric network is modeled using 3D MHD numerical simulations (Bingert, 2009). The equations for mass, momentum and energy conservation (see section 2.2.2) are solved using the Pencil Code (Brandenburg and Dobler, 2002), a highly modular code for astrophysical fluid dynamics with magnetic fields. It applies sixth-order finite differences for spatial derivates and a cache-efficient third-order Runge-Kutta scheme (Williamson, 1980) for explicit time-stepping. Further details can be found in Dobler and Brandenburg (2001) and in Bingert (2009).

For the coronal simulations specific physics modules and boundary conditions (section 2.1.5) are included in the Pencil Code. In the model proposed the heating of the corona is due to the dissipation of currents which result from the stressing
of the magnetic field by horizontal motions in the photosphere, similar to the approach by Gudiksen and Nordlund (2005a, 2005b). The horizontal motions at the lower boundary, i.e. the photosphere, with properties close to solar granulation are computer-generated (Gudiksen and Nordlund, 2002).

2.1.2 Influence of Microphysics

Averaging methods inherent to the MHD approximation (section 2.2) do not allow to analyze processes that happen on small scales (below the resolution limit of the simulation) in the corona. As an example, the nanoflare scenario (section 1.1.3.2) results in the creation of discontinuities in the magnetic field that cannot be resolved in current 3D MHD simulations of the corona. One should therefore be aware of the fact that the microphysics of dissipation is an approximation in these models. In the numerical description of the 3D MHD model applied in this work, plasma heating is due to Ohmic dissipation of currents. The Ohmic dissipation term \( \eta j^2 \) (\( j^2 \) squared current density, \( \eta \) magnetic diffusivity) is a source term in the energy equation (section 2.2.2.5). On the Sun, \( \eta \) is very small and magnetic field gradients must become large before dissipation occurs. Since the resolution of the simulation cannot be sufficient to resolve the current sheets that form in nature, we operate with an \( \eta \)-value many orders of magnitude larger than on the Sun, so that dissipation starts at much smaller magnetic field gradients. However, the total amount of energy flux that is heating the corona is independent of the exact physical process that thermalizes the Poynting flux, and thus the energy deposited in the corona stays constant (Galsgaard and Nordlund, 1996). The spatial and temporal scales involved in the dissipation of this flux depend on microphysical details that are not subject of this work.

2.1.3 The Setup of the Simulation

The success of previous coronal models (Gudiksen and Nordlund, 2005a, 2005b) and the good match of the model emission and observations (Peter et al., 2004, 2005, 2006) greatly motivated this work. In the original models of Gudiksen & Nordlund an active region with one sunspot was scaled down spatially by a factor of five to fit into the computational domain. By selecting this approach, the chromospheric network which is a major source of magnetic flux on the solar surface and an important indicator of stellar activity is suppressed in the authors’ simulations. An appropriate model of an active region should include the chromospheric network, too. Therefore, a generic magnetogram of an active region was generated that allows to study the interaction of magnetic structures on small scales Bingert (2009). This model served as a key tool for the current work. The active region magnetic field is identical to the one used by Gudiksen and Nordlund (2002), i.e. a spatially downscaled magnetogram of Active Region 9114 observed with the Michelson Doppler Imager.
2.1 MHD Simulations of the Corona in 3D with the Pencil Code

(MDI) onboard the Solar and Heliospheric Observatory (SOHO) on August 8, 2000 (Fig. 2.2). A downscaling was performed in order to fit the active region into the computational domain (50x50 Mm² horizontally). To highlight the interaction of the active region with the chromospheric network, the quiet Sun flux density was enhanced by a factor of four.

The left panel of Fig. 2.1 shows the vertical component of the magnetic field of the spatially scaled down active region including two main polarities and no magnetic network as obtained by Gudiksen and Nordlund (2002) after the start-up phase of their simulation. The right panel of Fig. 2.1 shows the magnetically complex region consisting of the scaled down active region plus enhanced magnetic network as calculated by Bingert (2009).

![Image: Initial magnetogram without magnetic network as obtained by Gudiksen and Nordlund (2002) after the start-up phase of their simulation. Right: Magnetogram with enhanced magnetic network flux as calculated by Bingert (2009) at timestep t=0 min of the simulation, i.e. after the start-up phase.]

2.1.4 Computational Domain

In order to resolve the photospheric motions that eventually lead to the braiding of magnetic field lines, the spatial resolution in the simulation box has to be at least half the size of a granule, i.e. 300-400 km. The computational domain should contain at least one super-granular cell (≈ 20 Mm) surrounded by another cell in each direction in order to avoid boundary effects. These requirements are accomplished by using 256 grid points horizontally in a box of 50x50 Mm². Large loops connecting regions of high magnetic activity can reach heights of up to 20 Mm, therefore the height of the box should be around 30 Mm. Previous one-dimensional models Müller (2004)
have shown that the vertical resolution in the transition region should be of the order of a few tens of kilometers which can be obtained by using a non-uniform grid with 250 grid points in the vertical direction.

2.1.5 Initial Conditions and Boundary Conditions for the Simulation Runs

The following summarizes the most important initial and boundary conditions for the corona-in-a-box simulation according to Bingert (2009).

- A standard stratified atmosphere (VAL-C; Vernazza et al., 1981) is being used as an initial condition for the temperatures and densities in the simulation box.
2.1 MHD Simulations of the Corona in 3D with the Pencil Code

- A SOHO/MDI high-resolution magnetogram is used at the lower boundary. This implies periodic boundary conditions for all other quantities in the horizontal direction.

- The magnetic field at the start of the simulation is obtained by a potential field extrapolation. Its temporal evolution is determined by the induction equation. At the bottom layer the magnetic diffusivity \( \eta \) is set to zero and the field lines move with the plasma.

- The observed solar granulation velocity pattern is reproduced by a weighted Voronoi tesselation method \((\text{Gudiksen and Nordlund}, 2002)\) and a Poynting flux is generated in the presence of a magnetic field. This photospheric driver twists and braids the magnetic field lines and therefore provides a large reservoir of mechanical energy.

- In order to stabilize the active region during the simulation that would otherwise diffuse and loose its structure a fraction of the original magnetic field is constantly added to the vertical field at each timestep.

- The energy transport from the photosphere to the upper chromosphere is not treated properly by the code, e.g. radiative transport is not included in detail. Instead a standard height-dependent temperature profile is applied for this region based on Newton’s law of cooling.

- No mass can pass through the upper layer of the simulation box. The temperature and density of the top layer can change, but temperature and density gradients are set to zero at the upper boundary layer.

- The initial start-up period is approximately 30 minutes for the short run and approximately one hour for the long run. This time span assures that enough current density is built up in order to sustain a million degree corona based on an energy balance between heating, radiative losses and heat conduction. After the initial start-up period the system is able to evolve independently of the initial conditions imposed at the very beginning of the simulation. This is when the time series is being started. From now on this time is referred to as \( t=0 \) min of the simulation.

The simulation results that are analyzed in this study were provided by Sven Bingert \((\text{Bingert}, 2009)\). The calculations were carried out on the computer cluster of the Kiepenheuer Institut für Sonnenphysik (KIS) in Freiburg. The calculations for the 30 minute high-resolution run took 35 days using all of the cluster’s 64 CPUs. One timestep of this simulation corresponds to 0.5 ms solar time and required about 0.84 s on the KIS linux cluster. For the 71 minute low-resolution run approximately 20 days of calculations were required. In this simulation it took 0.2 s to calculate one timestep that corresponds to 1 ms solar time.
2.2 Basic MHD Equations

In this chapter equations are discussed that govern the dynamics of magnetized plasma in both the solar transition region and corona.

2.2.1 Maxwell’s Equations

The magnetic field in the transition region and corona is governed by

Ampere’s law
\[ \nabla \times \mathbf{B} = \mu \mathbf{j}, \quad (2.1) \]

Faraday’s law
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.2) \]

Gauss’ law
\[ \nabla \cdot \mathbf{B} = 0, \quad (2.3) \]

and Ohm’s law
\[ \mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (2.4) \]

where \( \mathbf{E} \) [V/m] is the electric field, \( \mathbf{j} \) [A/m\(^2\)] is the current density, \( \mathbf{v} \) [m/s] is the plasma velocity and \( \mathbf{B} \) [T] is the magnetic induction. In solar physics magnetic induction values are often quoted in Gauss (1 G = 10\(^{-4}\) T) and large distances are conveniently expressed in Mm (1 Mm = 10\(^6\) m). The terms \( \mu = 4\pi \times 10^{-7} \) N A\(^{-2}\) and \( \sigma \) are the magnetic permeability and the electrical conductivity. The displacement current has been neglected in Ampere’s law (Eq. 2.1), since electromagnetic variations are considered non-relativistic in magnetohydrodynamics [Priest, 1984].

2.2.1.1 The Induction Equation

Ohm’s law written in the form of Eq. 2.4 is a relation between electric current density, electric field, magnetic field and plasma velocity. In the absence of magnetic fields Eq. 2.4 reduces to \( \mathbf{j} = \sigma \mathbf{E} \) stating that the electric field is proportional to the current density. An equivalent formulation is \( I = V/R \), the current \( I \) through a resistance \( R \) equals the potential difference \( V \) measured across the resistance divided by the resistance of the conductor. The second term in Eq. 2.4 arises from the plasma moving at a non-relativistic speed in the presence of a magnetic field, thus being subject to an electric field.

Applying Ampere’s law to Eq. 2.4 yields
\[ \mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{1}{\mu \sigma} \nabla \times \mathbf{B} \quad (2.5) \]
2.2 Basic MHD Equations

for the electric field in the laboratory frame. Application of the vector identity
\[ \nabla \times (\nabla \times \mathbf{B}) = \nabla (\nabla \cdot \mathbf{B}) - (\nabla \cdot \nabla) \mathbf{B}, \]  
(2.6)
as well as of Gauss’ law (2.3) leads to the induction equation
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \]  
(2.7)
with the magnetic diffusivity \( \eta = 1/(\mu \sigma) \). For a given velocity \( \mathbf{v} \) the induction equation determines the evolution of the magnetic field with time. Typical values for \( \eta \) in the chromosphere and corona are \( 8 \times 10^8 \, T^{-3/2} \) and \( 10^9 \, T^{-3/2} \) m² s⁻¹ (Priest, 1984). The induction equation (2.7) can also be expressed as
\[ \frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} - \eta \mu \mathbf{j}, \]  
(2.8)
where the magnetic vector potential \( \mathbf{A} \) (\( \mathbf{B} = \nabla \times \mathbf{A} \)) is used which eliminates the problem of keeping the magnetic field divergence-free (Eq. 2.3).

2.2.1.2 Diffusion, Advection and “Frozen-In” Approximation

The induction equation (2.7) is made up by both a conductive term and a diffusive term, the ratio of which is a dimensionless parameter called the magnetic Reynolds number. Assuming a typical scale length \( l_0 \) for the variation of \( B \) and a typical plasma speed \( V_0 \) the magnetic Reynolds number can be expressed as
\[ R_m = \frac{l_0 V_0}{\eta}. \]  
(2.9)
The magnetic Reynolds number determines the strength of the coupling between the magnetic field and the plasma flow. If \( R_m \ll 1 \), the time-dependent behavior of the magnetic flux density is governed by the diffusive equation
\[ \frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}. \]  
(2.10)
Generally, \( R_m \gg 1 \) applies to the solar atmosphere, as the spatial scales are large compared to the motions and \( \eta \approx 1 \). In the large magnetic Reynolds number limit the induction equation reduces to
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \]  
(2.11)
In this case (ideal MHD) the “frozen-in” approximation is applicable.
"Frozen-in" Approximation  Alfvén’s theorem states that the magnetic flux

\[ \Phi = \int_F \mathbf{B} \cdot d\mathbf{f} \quad (2.12) \]

through a surface \( F \) with boundary \( \partial F \) that is moving with the fluid is conserved. \( d\mathbf{f} \) is an element of the surface \( F \). Thus, the total time derivative of \( \Phi \) vanishes,

\[ \frac{d\Phi}{dt} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left[ \int_{F(t+\Delta t)} \mathbf{B}(t+\Delta t) \cdot d\mathbf{f} - \int_{F(t)} \mathbf{B}(t) \cdot d\mathbf{f} \right] \]

\[ = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left[ \int_{F(t+\Delta t)} \left[ \mathbf{B}(t+\Delta t) - \mathbf{B}(t) \right] \cdot d\mathbf{f} \right] \]

\[ = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left[ \int_{F(t+\Delta t)} \mathbf{B}(t) \cdot d\mathbf{f} - \int_{F(t)} \mathbf{B}(t) \cdot d\mathbf{f} \right] \]

\[ = \int_F \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{f} - \int_{\partial F} (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{s} \]

\[ = \int_F \left[ \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) \right] \cdot d\mathbf{f} = 0. \quad (2.14) \]

Here, \( d\mathbf{s} \) is an element of the boundary \( \partial F \) of \( F \). The area elements \( -\int_{F(t)} d\mathbf{f}, \int_{\partial F} d\mathbf{s} \times \mathbf{v} \Delta t \) and \( \int_{F(t+\Delta t)} d\mathbf{f} \) form, in the limit \( \Delta t \to 0 \), a closed surface, so that \( \nabla \cdot \mathbf{B} = 0 \) leads to

\[ \int_{F(t)} \mathbf{B} \cdot d\mathbf{f} - \int_{F(t+\Delta t)} \mathbf{B} \cdot d\mathbf{f} + \Delta t \int_{\partial F} \mathbf{B} \cdot (d\mathbf{s} \times \mathbf{v}) = 0. \quad (2.15) \]

Thinking of a magnetic flux tube as a volume enclosed by a set of field lines that intersect a simple closed curve, this means that the plasma within a magnetic flux tube remains always in that flux tube, as the plasma moves. The magnetic field lines behave as if they move with the plasma and one refers to them as being “frozen into the plasma”. Diffusion of magnetic field lines relative to the plasma can be neglected and changes in \( \mathbf{B} \) are determined entirely by the transport of field lines into and out of the plasma volume through plasma motions of velocity \( \mathbf{v} \).

2.2.2 Dynamics of Solar Plasmas

The outer layers of the Sun and stars in general are made up of matter in an ionized state - a plasma. A plasma can be identified as any state of matter that contains enough free charged particles for its dynamics to be dominated by electromagnetic forces. Thermonuclear burning in stars is the source of plasmas in space that account for 99% of the matter in the visible universe.
One of the most notable features of a plasma is its ability to maintain a state of charge neutrality. One of the fundamental plasma parameters is the Debye length

\[ \lambda_D = \left( \frac{\epsilon_0 k_B T}{n_e e^2} \right)^{1/2} = 69.0 \left( \frac{T}{n_e} \right)^{1/2}. \]  \hspace{1cm} (2.16)

This length is a measure of the distance over which the electric field of a charged particle is shielded by the random thermal motions of the other charged particles. \( \epsilon_0 \approx 8.854 \times 10^{-12} \text{ F m}^{-1} \) is the vacuum dielectricity. The Debye sphere \( \Lambda = \frac{4}{3} \pi n_e \lambda_D^3 \) is defined in terms of the Debye length. \( \Lambda \) is also called the plasma parameter. The Debye shielding picture is valid provided there are enough particles in the Debye sphere, i.e. \( \Lambda \gg 1 \). A major consequence of this condition is that a plasma can be treated as a collection of independent fluid elements described by a distribution function that evolves under the influence of local forces and collisions. Associated with the Debye length scale is the inverse time scale

\[ \omega_{pe} = \left( \frac{n_e e^2}{m_e \epsilon_0} \right)^{1/2} \]  \hspace{1cm} (2.17)

known as the plasma frequency. \( n_e \) is the electron density, \( e \) is the electron charge and \( m_e \) is the electron mass. When the plasma frequency is large compared to the electron-neutral collision frequency \( (\nu_{en} = 1/\tau_{en}) \), electrostatic interactions dominate. The three criteria for the existence of the plasma state can be summarized as

- \( \Lambda \gg 1 \) (plasma parameter large)
- \( \lambda_D \ll L \) (Debye length small compared to typical length scales \( L \))
- \( \omega_{pe} \tau_{en} > 1 \) (low neutral collisionality)

Typical values for the plasma parameter in the simulation are \( 10^6 - 10^8 \) in the corona. The Debye length in the corona is in the millimeter range, and the resolution of the simulation grid varies between 75 km and 285 km (Fig. 2.3). As shown in the figure, the electron collisional mean free path is larger than the Debye length, i.e., \( \omega_{pe} \tau_{en} > 1 \). Therefore, the criteria for a plasma according to the description above are fulfilled in the simulation.

In order to completely describe the state of a plasma and characterize the electromagnetic field in the plasma, all the particle locations and velocities need be considered. Tracking all the plasma particles is inconvenient and can often be omitted. Less detailed classes of descriptions are commonly used, e.g., kinetic models as a statistical approach or fluid models as a magnetohydrodynamic (MHD) approach. In the fluid model, the plasma is described in terms of macroscopic parameters such
2 Theory and Methodology

Figure 2.3: Electron mean free path $\lambda_e = 1.07 \times 10^9 \frac{T[K]}{n[m^{-3}] \ln \Lambda}$ (Boyd and Sanderson, 2003). Debye length $\lambda_D$ and the typical magnetohydrodynamic length scale (vertical grid spacing) as a function of height in the simulation box. Dashed lines indicate minimum and maximum values of the electron mean free path.

as density, pressure and velocity of “physically small” fluid elements that contain many plasma particles. Fluid models are often accurate when collisionality is sufficiently high to keep the plasma velocity distribution close to a Maxwell-Boltzmann distribution. Magnetohydrodynamics, a simple fluid model, is treating the plasma as a single fluid governed by a combination of Maxwell’s equations and the Navier-Stokes equations. The plasma is described in terms of a single flow velocity at a defined temperature at each spatial location.

In the MHD approximation, the behavior of a plasma is determined by a simplified form of Maxwell’s equations (section 2.2.1), together with Ohm’s law (Eq. 2.4), an equation of state (section 2.2.2.2) and equations of mass continuity (section 2.2.2.3), motion (section 2.2.2.4) and energy (section 2.2.2.5).

2.2.2.1 Plasma-$\beta$

A useful parameter to distinguish whether hydrodynamic or magnetic forces govern the dynamics of the plasma is the so-called plasma-$\beta$, the ratio of plasma pressure $p_0 = nk_BT$ and magnetic pressure $p_m = B^2/2\mu_0$,

$$\beta = \frac{p_0}{p_m} = \frac{2\mu_0 nk_BT}{B^2} \approx 0.03 \frac{n [10^{15} m^{-3}] T [10^6 K]}{B^2 [10 G]}.$$ (2.18)
2.2 Basic MHD Equations

<table>
<thead>
<tr>
<th>Domain</th>
<th>n [m$^{-3}$]</th>
<th>T [K]</th>
<th>$B$ [G]</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photosphere</td>
<td>$10^{23}$</td>
<td>$6 \times 10^4$</td>
<td>1-1500</td>
<td>&gt;10</td>
</tr>
<tr>
<td>Chromosphere</td>
<td>$10^{19}$</td>
<td>$2 \times 10^4 - 10^4$</td>
<td>10-100</td>
<td>10-0.1</td>
</tr>
<tr>
<td>Transition Region</td>
<td>$10^{15}$</td>
<td>$10^4 - 10^6$</td>
<td>1-10</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Corona</td>
<td>$10^{14}$</td>
<td>$10^6$</td>
<td>1-10</td>
<td>$10^{-2.1}$</td>
</tr>
</tbody>
</table>

| Table 2.1: Basic parameters for domains in the solar atmosphere according to Schrijver and Siscoe (2009). Densities $n$, temperatures $T$, magnetic field strength $B$ and plasma-$\beta$ of the different layers are listed. |

In this term, $\mu_0 = 4\pi \times 10^{-7}$ N A$^{-2}$ is the magnetic vacuum permeability, $B$ is the magnetic field strength, $k_B = 1.38 \times 10^{-23}$ m$^2$ kg s$^{-2}$ K$^{-1}$ is the Boltzmann constant, $n$ denotes the particle density, and $T$ is the temperature of the plasma. For $\beta \ll 1$, magnetic forces dominate over the gas forces and the magnetic field determines the dynamics of the plasma (Table 2.1). If, on the other hand, $\beta \gg 1$, plasma forces dominate over the magnetic forces and the magnetic field is advected by the plasma.

A kinetic plasma-$\beta$ can be defined using the kinetic energy density $e_{\text{kin}} = \frac{1}{2} \rho v^2$,

$$\beta_{\text{kin}} = \frac{e_{\text{kin}}}{\rho m_p} = \frac{\mu_0 \mu m_p n v^2}{B^2} \approx 10^{-6} n \frac{[10^{15} \text{ m}^{-3}] v^2 [10^3 \text{ m/s}]}{B^2 [10 \text{ G}]}$$

(2.19)

where the mass density $\rho$ has been substituted by the product of mean particle mass $m$ ($m = \mu m_p$, $\mu$ is the mean atomic weight, and $m_p$ is the proton mass) and particle density $n$.

2.2.2.2 The Equation of State

The equation of state can be expressed in the form of the perfect gas law

$$p = \frac{R}{\mu} \rho T,$$

(2.20)

in which $R$ is the specific gas constant, $\rho$ is the mass density, and $\mu$ is the mean atomic weight. Replacing $R$ by the ratio of the Boltzmann constant $k_B$ and the proton mass $m_p$, Eq. (2.20) results in

$$p = k_B \frac{\rho T}{m_p}.$$

(2.21)

In this formulation the mean particle mass $m$ is defined as $\mu m_p$. Using the total number of particles per unit volume $n$, the ideal gas law can be written

$$p = n k_B T,$$

(2.22)

29
in which the mass density $\rho$ has been replaced by $mn$.

### 2.2.2.3 The Continuity Equation

The continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

(2.23)

or

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v}.$$  

(2.24)

$\rho$ denotes the plasma density, $\mathbf{v}$ the plasma velocity, $t$ the time and $D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$ is the convective derivative. The continuity equation can also be formulated using the term $\ln \rho$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{v},$$

(2.25)

which is the Pencil code formulation of the continuity equation.

### 2.2.2.4 The Equation of Motion

The equation of motion can be written in the form

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \nabla \Phi_{\text{grav}} + \mathbf{F}_v,$$

(2.26)

where $\rho$ is the mass density, and $p$ is the plasma pressure. The equation indicates that the plasma motion is subject to the pressure gradient $\nabla p$, the Lorentz force $\mathbf{j} \times \mathbf{B}$ per unit volume (see below), and to the effects of gravity $\nabla \Phi_{\text{grav}} = \frac{\rho M(r)}{2\pi \rho r^2} \hat{r}$ (where $M(r)$ is the mass of the Sun inside a radius $r$ and $G$ is the gravitational constant) and viscosity $\mathbf{F}_v$ (see below).

If only the pressure gradient is effective, the resulting equation of motion is called Euler’s equation

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p.$$  

(2.27)

Euler’s equation describes the motion of an ideal fluid in which no dissipation of kinetic energy into heat takes place by viscosity, nor is heat transported by thermal conduction or radiation. When the velocity is constant in time ($\frac{\partial \mathbf{v}}{\partial t} = 0$), the flow is steady at each location. In the special case $\mathbf{v} = 0$ the hydrostatic equilibrium relation

$$\nabla p = -\rho \mathbf{g}$$

(2.28)

is obtained.
2.2 Basic MHD Equations

**The Lorentz Force** An electric current is generated when plasma moves in a magnetic field. This current of density $j$ interacts with the magnetic field and causes a magnetic volume force that opposes the motion. The Lorentz force $j \times B$ is directed across the magnetic field. It can be separated into a magnetic tension force $(B \cdot \nabla)B/\mu$ and a magnetic pressure force $\nabla(B^2/(2\mu))$, thus reflecting two effects. The Lorentz force shortens magnetic field lines through the tension force and also compresses plasma through the pressure term.

**The Viscous Force** In real fluids, the transfer of momentum occurs by the transport of fluid volumes with different velocities, expressed by the advective term $(\mathbf{v} \cdot \nabla)\mathbf{v}$ in Euler’s equation. Additional momentum transfer is caused by the internal friction that is due to collisions between particles moving with adjacent layers of the fluid at different velocities. If there are no velocity gradients within the flow, the viscous forces will disappear. A general expression for the viscous force is

$$\mathbf{F}_\nu = 2\nu \mathbf{S} \nabla \rho,$$

(2.29)

where $\nu$ is the kinematic viscosity and $\mathbf{S}$ is the traceless rate-of-strain tensor

$$S_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{v} \right).$$

(2.30)

A more familiar notation for the viscous force is

$$\mathbf{F}_\nu = \rho \nu \left( \nabla^2 \mathbf{v} + \frac{1}{3} \nabla(\nabla \cdot \mathbf{v}) + \frac{2}{\rho} \mathbf{S} \nabla \rho \right).$$

(2.31)

The identity of the expressions for the viscous force of both (2.29) and (2.31) is shown in appendix B. For a fully-ionized hydrogen plasma $\rho \nu = 2.21 \times 10^{-16} \frac{T^{5/2}}{\ln \Lambda}$ kg m$^{-1}$ s$^{-1}$ (Spitzer, 1962), where $\ln \Lambda$ is the Coulomb logarithm, the tabulated values of which range between 20 in the corona and 10 in the chromosphere (Foukal, 2004).

**Force-free Fields** In the chromosphere and corona, the plasma is in a so-called force-free equilibrium. Plasma frozen to the magnetic field is constrained to move along the magnetic field lines, for which

$$j \times B = 0$$

(2.32)

holds. The magnetic fields that are anchored in the photosphere and fan out in the transition region and corona give rise to the field-aligned appearance of motions in fibrils, spicules, loops and prominences.
2 Theory and Methodology

The particular case when $j = 0$ is called current-free or potential field. When using Ampere’s law (Eq. 2.21), Eq. 2.32 implies

$$\nabla \times \mathbf{B} = \alpha \mathbf{B},$$

(2.33)

where $\mathbf{B} \cdot \nabla \alpha = 0$. This means that the field line twist parameter $\alpha(x)$ is constant along each magnetic field line. When $\alpha$ takes the same value on each field line, the magnetic field is called linear or constant-$\alpha$ field.

2.2.2.5 The Energy Equation

The energy equation can be expressed in many different forms, e.g. in terms of the inner energy $e = \frac{p}{(\gamma - 1)p} = \frac{3}{2} p$ of a closed thermodynamic system,

$$\rho \frac{De}{Dt} + p \nabla \cdot \mathbf{v} = \eta \mu_0 j^2 + 2 \nu \rho S^2 + \zeta \rho (\nabla \cdot \mathbf{v})^2 - n_e n_H Q(T) - \nabla \cdot \mathbf{q}.$$  

(2.34)

In 2.34, $\eta \mu_0 j^2$ is the Ohmic dissipation term, $S$ is the traceless rate-of-strain tensor, $\zeta$ describes the bulk viscosity, $\mathbf{q}$ is the heat flux due to particle conduction, and $n_e n_H Q(T)$ describes the radiative loss for an optically thin atmosphere. The plasma is assumed to be a fully ionized hydrogen gas, thus the ratio of specific heats is $\gamma = 5/3$. Terms of the energy equation are discussed in the following sections.

Viscous Heating  The heat dissipation per unit volume can be expressed in terms of the local rate of shear $S_{ij}$,

$$H_v = 2 \nu \rho S^2.$$  

(2.35)

The shear term is discussed in section 2.2.2.4.

Radiative Loss Function  In the optically thin part of the solar atmosphere, the radiative loss is no longer coupled to the radiation field by the radiation transfer equation. Rather, its form is similar to the volume emissivity of an optically thin line (Eq. 2.45)

$$\nabla \cdot \mathbf{F}_{\text{rad}} = n_e n_H Q(T) \sim n_e^2 Q(T).$$  

(2.36)

The radiative loss function $Q(T)$ is expressing the temperature dependence and requires the knowledge of all the lines and continua that contribute to radiation at a given temperature, density and chemical composition. An analytic approximation is $Q(T) = \chi T^\alpha [\text{W/m}^3]$, with the piecewise constants $\chi(T)$ and $\alpha(T)$. For the numerical calculations reference is made to the radiative loss function of Cook et al. (1981) shown in Fig. 2.4. Important qualitative features of this function are the broad peak around 10^5 K, which is due to the strong resonance lines of highly ionized states of elements such as carbon, nitrogen, oxygen and iron and the decrease towards coronal temperatures. From Eq. 2.36 it can be seen that the radiation increases with the particle density squared.
2.2 Basic MHD Equations

Thermal Conduction  The conductive loss term is expressed as the divergence of the conductive flux $q$. The heat flux due to thermal conduction is given by

$$ q = -\kappa \nabla T, \quad (2.37) $$

where $\kappa$ is the thermal conductivity. According to Spitzer (1962), the thermal conductivity in the direction parallel to the magnetic field lines is

$$ \kappa = 1.8 \times 10^{-10} \frac{T^{5/2}}{\ln \Lambda} \, [\text{W m}^{-1} \, \text{K}^{-1}] \quad (2.38) $$

in a fully ionized plasma. The conductivity strongly depends on the temperature, but is only marginally influenced by the electron density through the Coulomb logarithm $\ln \Lambda$ (see Eq. 2.31). At low photospheric temperatures, the $T^{5/2}$ dependence of $\kappa$ causes energy transport by thermal conduction to be negligible for most purposes. The combination of 2.37 and 2.38 results in the expression of the conductive loss term

$$ \nabla \cdot q = -\nabla \cdot (\kappa \nabla T). \quad (2.39) $$

The steep temperature gradient in the solar transition region leads to a large flux of thermally conducted heat in this layer.
2.3 Emission Line Spectroscopy

In hot plasmas like the plasma of the solar transition region and corona, many different atomic processes can excite and deexcite energy levels of an ion and give rise to ionization and recombination. Atomic processes important for the solar transition region and corona are discussed in this section. A description on how to calculate optically thin emission lines from the atomic parameters will follow at the end of this section.

2.3.1 Atomic Processes in the Transition Region and Corona

Not all of the known atomic processes which can change the states of ionization and excitation of atoms and ions are equally important in the hot plasma of the solar transition region and corona. Collisional excitation is the most common excitation mechanism in the solar transition region and corona, whereas only few ions are excited radiatively due to the relatively weak solar radiation field at UV wavelengths. For the same reason stimulated emission is negligible and photoionization is not very efficient. Spontaneous radiative decay is the fastest deexcitation process and much more efficient in depopulating excited levels than are collisional deexcitation processes. Most ions in the solar transition region can only be ionized by collisions since the UV radiation field is too weak to make photoionization efficient. Radiative recombination is the dominating recombination mechanism. At the low densities of the solar transition region and corona, the rate for three-body recombination is negligible. Autoionization and dielectronic recombination can be important for the ionization state of the plasma, but are only effective at higher temperatures.

The main processes which can change the states of ionization and excitation of atoms and ions in the transition region and corona are listed below. The symbol \( X^{q+} \) refers to a \( q \)-times ionized atom. Excited states are denoted by an asterisk, e.g. \([X^{q+}]^\ast\).

*Collisional excitation* is induced by an electron \( e^- \) impacting on ion \( X^{q+} \)

\[
X^{q+} + e^- \rightarrow [X^{q+}]^\ast + e^-.
\]  
(2.40)

*Collisional ionization* occurs by collisions of ions with free electrons, when an orbiting electron of the ion is removed, and the ion is left in the next higher ionization state

\[
X^{q+} + e^- \rightarrow X^{(q+1)+} + 2e^-.
\]  
(2.41)

*Radiative recombination* (Photorecombination) is the capture of an electron into the excited state of an ion where the excess energy is removed by the emission of a photon of energy \( h\nu \)

\[
X^{q+} + e^- \rightarrow X^{(q-1)+} + h\nu.
\]  
(2.42)
2.3 Emission Line Spectroscopy

Dielectronic Recombination is a two-step resonance process involving two electrons. A free electron is captured by an ion, simultaneously a bound electron of the ion becomes excited, resulting in a double excitation of the ion plus a subsequent radiative deexcitation. This process is only possible if the ion \( X^{q+} \) has at least one bound electron.

Autoionization is an alternative decay mode of the doubly excited state. An ion is initially in a doubly excited state and spontaneously ionizes (autoionizes) leaving an ion and a free electron.

Spontaneous radiative decay is a process which depopulates excited levels. An electron spontaneously falls from a higher to a lower energy state by the emission of a photon with energy \( h\nu \)

\[
[X^{q+}]^* \rightarrow X^{q+} + h\nu. \tag{2.43}
\]

The rate for this process is given by \( n_j A_{ji} \), where \( n_j \) denotes the population of an atomic energy level \( j \) and \( A_{ji} \) is the Einstein coefficient for a spontaneous transition from level \( j \) to a lower level \( i \).

Table 2.2 is listing the rates at which the atomic processes take place in the solar transition region and corona, together with typical timescales. The numbers illustrate that the typical timescales for ionization and recombination processes are of the order of tens to hundreds of seconds which is much longer than the timescales for excitation processes.

2.3.2 Formation of Optically Thin Emission Lines

Each atomic species contained in the coronal plasma has many different bound energy levels. For any upper level \( j \) and lower level \( i \) the probability for spontaneous emission of a photon of energy \( h\nu \) per unit time is \( A_{ji} \). If the number density of atoms in the excited level is \( n_j \), then the volume emissivity of the plasma in the \( j \) to \( i \) transition is

\[
\epsilon_\nu = h\nu j_i \cdot n_j \cdot A_{ji} \cdot \psi_\nu, \tag{2.44}
\]

where \( \psi_\nu \) is the emission line profile, normalized to unity when integrated over all frequencies. Units of \( \epsilon_\nu \) are \( \text{W m}^{-3} \text{ Hz}^{-1} \). In general, there is interest to know the total emissivity of the transition

\[
\epsilon_{ji} = h\nu_{ji} \cdot n_j \cdot A_{ji}, \tag{2.45}
\]

which has units of \( \text{W m}^{-3} \). In order to relate the emissivity to the actual observed intensity of a line, the power of an observed line is defined as

\[
P_{ji} = \int_V \epsilon_{ji} \, dV, \tag{2.46}
\]
where $dV$ is a volume of plasma with temperature $T$ and particle density $n$. Units of $P_{ji}$ are Watt.

The flux $F_{ji}$ per unit time at distance $d$ from a volume of plasma is

$$F_{ji} = \frac{\int_V \varepsilon_{ji} dV}{4\pi d^2}, \quad (2.47)$$

where $d$ is the distance between the star and the observer.

For the purpose of this study it is convenient to express the number density of ions in the excited level $j$ in terms of the relative population of the excited level $n_j/n_{ion}$, the relative abundance of the ionic species $n_{ion}/n_{el}$, the abundance of the element relative to hydrogen $n_{el}/n_H = A_{el}$ and the number density of hydrogen atoms relative to the number density of electrons $n_H/n_e$. This can be expressed as

$$n_j = \frac{n_j}{n_i \cdot n_e} \cdot \frac{n_i}{n_{ion}} \cdot \frac{n_{ion}}{n_{el}} \cdot \frac{n_{el}}{n_H} \cdot \frac{n_H}{n_e} \cdot n_e^2. \quad (2.48)$$

Inserting this relation into Eq. $(2.47)$ gives

$$F_{ji} = \frac{1}{4\pi d^2} \int_V h\nu_{ji} \cdot \frac{n_j}{n_i \cdot n_e} \cdot \frac{n_i}{n_{ion}} \cdot \frac{n_{ion}}{n_{el}} \cdot \frac{n_{el}}{n_H} \cdot \frac{n_H}{n_e} \cdot n_e^2 \cdot A_{ji} \, dV. \quad (2.49)$$

This equation comprehends all the parameters necessary to calculate the total flux of an optically thin spectral line at a given distance $d$. The excitation, ionization
and abundance terms of Eq. 2.49 will be discussed in sections 2.3.2.2, 2.3.2.3 and 2.3.2.4 but some selected aspects need to be discussed at this point.

In the coronal approximation that will be discussed in section 2.3.2.2, collisional and radiative coupling of the respective level is possible with the ground level \( n_0 \) only and \( n_0 \approx n_{\text{ion}} \). The relative population of the excited level is mostly a function of temperature, as will also be explained in section 2.3.2.2. The same is true for the relative abundance of the ionic species if ionization equilibrium is assumed (see section 2.3.2.3). If the emitting region is considered to be a uniform, spherical shell of thickness \( h \), i.e. \( dV = 4\pi R^2 dh \) (\( R \) is the radius of the star), the expression for the flux will be

\[
F_{ji} = \frac{1}{d^2} \int_V G(T) \cdot n_e^2 \, dV = \frac{4\pi R^2}{d^2} \int_h G(T) \cdot n_e^2 \, dh,
\]

(2.50)

where \( d \) is the distance of the star. The temperature-dependent terms in Eq. 2.50 have been grouped together into the so-called contribution function

\[
G(T) = \frac{h \nu_{ji}}{4\pi} \cdot A_{ji} \cdot \frac{n_j}{n_e} \cdot \frac{n_{\text{ion}}}{n_{\text{el}}} \cdot \frac{n_{\text{el}}}{n_H} \cdot \frac{n_H}{n_e}.
\]

(2.51)

Various definitions of contribution functions can be found in the literature. For resonance lines the contribution function \( G(T) \) is a function of temperature only. The dependence of \( G \) on temperature results from both the ionization equilibrium term \( n_{\text{ion}}/n_{\text{el}} \) and from the relative population of the excited level \( n_j/n_{\text{ion}} \) (section 2.3.2.2, Eq. 2.60). When forbidden lines or intercombination lines are considered, \( G(T, n_e) \) depends remarkably on density due to the collisional population and de-population processes determining the \( n_j/n_{\text{ion}} \) term (see section 2.3.2.3).

The temperature at which the contribution function of the line under consideration is reaching a maximum, is commonly defined to be the line formation temperature \( T_f \) (for further details see section 2.3.2.2). Due to the high temperature sensitivity of the relative ion abundance the line formation temperature almost coincides with the maximum of ionization temperature. However, if steep temperature gradients are present, or if He-like ions are considered that are present over a relatively broad temperature range, this assumption might be insufficient or even incorrect. Table 2.3 is listing the formation temperatures of several transition region and coronal lines that will be studied in this work along with their atomic transitions and wavelengths.

---

1 Sometimes the element abundance factor is not included and the contribution function \( C(T, n_e) \) is defined by

\[
C(T, n_e) = \frac{h \nu_{ji}}{4\pi} \cdot A_{ji} \cdot \frac{n_j}{n_e} \cdot \frac{n_{\text{ion}}}{n_{\text{el}}} \cdot \frac{n_{\text{el}}}{n_H} \cdot \frac{n_H}{n_e}.
\]
2.3.2.1 The Differential Emission Measure Distribution

The temperature distribution of the corona can quantitatively be expressed by the so-called differential emission measure distribution (DEM). The DEM indicates the amount of plasma along the line of sight that is emitting radiation originating at temperatures between $T$ and $T + dT$. It is common to assume that the abundance of the element is constant over the source volume and therefore to define the DEM($T$) as

$$\text{DEM}(T) = n_e^2 \cdot \left( \frac{dT}{dh} \right)^{-1}. \quad (2.52)$$

The differential emission measure can be inserted into Equation 2.50 to result in

$$F_{ji} = \frac{4\pi R^2}{d^2} \int_T G(T) \cdot \text{DEM}(T) \, dT. \quad (2.53)$$

A mean value of $G(T)$ can be calculated and withdrawn from the integral when the temperature gradient along the line of sight is found to be sufficiently high, i.e. when the range of line formation is small compared to the range of density variations,

$$F_{ji} = \frac{4\pi R^2}{d^2} \langle G(T) \rangle \int_T \text{DEM}(T) \, dT. \quad (2.54)$$

In a similar way the flux can be expressed as

$$F_{ji} \approx \frac{1}{d^2} \langle G(T) \rangle \int_V n_e^2 \, dV \quad (2.55)$$

$$= \frac{1}{d^2} \langle G(T) \rangle \text{EM}_V, \quad (2.56)$$

with $\text{EM}_V$ being the volume emission measure

$$\text{EM}_V = \int_V n_e^2 \, dV. \quad (2.57)$$

The differential emission measure distribution can be directly obtained from the results of the MHD model (see section 3.3) or by inversion of Eq. 2.53. The observed quiet Sun differential emission measure distribution will be discussed in section 3.3.

2.3.2.2 Excitation and Deexcitation

It was pointed out in the previous section that in the low-density plasma of the outer solar atmosphere excitation and deexcitation processes generally take place on timescales significantly shorter than ionization and recombination processes. Therefore, the problem of calculating the population of excited levels can be separated
2.3 Emission Line Spectroscopy

from the problem of calculating the ionization balance. Considering the population of excited levels, any given level can be populated by collisional excitation from lower energy levels and by both collisional deexcitation and radiative decay from higher energy levels. The same level can be depopulated by collisional deexcitation to any higher level and by both collisional deexcitation and radiative decay to any lower level. The level population must be calculated solving the statistical equilibrium equation including the different processes.

Collisional transitions between two levels are governed by a rate coefficient \( C \), radiative transitions are governed by a rate coefficient \( A \). For each level in the ion, the rate equation reads

\[
\frac{dn_i}{dt} = \sum_{j \neq i} n_j n_e C_{ji} - n_i \sum_{j \neq i} n_e C_{ij} + \sum_{j > i} n_j A_{ji} - n_i \sum_{j < i} A_{ij}.
\]

(2.58)

The sum of the level populations must equal the ion number density

\[ n_{\text{ion}} = \sum_i n_i. \]

(2.59)

For a low-density plasma as found in the transition region and in the corona, it is assumed that the collisional excitation processes are faster than the ionization and recombination timescales, so that collisional excitation determines the population of excited states.

For allowed, electric dipole transitions the assumption can be made that the population of the upper level occurs mainly via collisional excitation from the ground state and that the spontaneous radiative decay dominates over all other depopulation processes. This assumption is also called the coronal model approximation. Based on this assumption, the statistical equilibrium equations (Eq. 2.58) reduce to the two-level atom approximation

\[ n_e n_0 C_{ij} = n_j A_{ji}, \]

(2.60)

where 0 denotes the ground and \( j \) the upper level. Since \( n_e C_{ij} \ll A_{ji} \), the population of the upper level is negligible compared to that of the ground level, and the population of the lower level can approximated by the total population of the radiating ion, \( n_0 \approx n_{\text{ion}} \).

The electron collision rate coefficient \( C_{ij} \) is obtained by integrating the cross-section for excitation by collisions \( \sigma_{ij} \) with electrons of velocity \( v \) over the electron velocity distribution

\[ C_{ij} = \int_{v_0}^{\infty} \sigma_{ij}(v) f(v) v \, dv, \]

(2.61)
where \( v_0 \) is the velocity that corresponds to the threshold energy of the transition. Collision cross-sections are commonly expressed in terms of the collision strength \( \Omega_{ij} \)

\[
\sigma_{ij} = \frac{\pi a_0^2 \Omega_{ij}(E)}{\omega_i E},
\]

(2.62)

where \( a_0 \) is the Bohr radius, \( \omega_i \) is the statistical weight of level \( i \), and \( E = \frac{1}{2}mv^2 \) is the kinetic energy of the exciting electron. Collision strengths are calculated quantum mechanically, and they are known for most ions of astrophysical interest.

### 2.3.2.3 Ionization

In order to determine the ionization state of the plasma the individual collisional and radiative ionization and recombination processes need to be considered in detail. For a hot low-density plasma as found in the transition region and corona the ion number density is determined by the competing processes of electron impact ionization and radiative and dielectronic recombination. Photoionization is not significantly impacting in the quiet solar atmosphere due to the weak radiation field.

The rate equation that describes the variation of the ion number density with time consists of the rate at which \( n_{z-1} \) is created by electron impact excitation from ion \( z - 1 \) and by recombination from ion \( z + 1 \) and as well as of the rate at which \( n_z \) is destroyed by ionization to ion \( z + 1 \) and by recombination to ion \( z - 1 \),

\[
\frac{dn_z}{dt} = n_e (n_{z-1} q_{z-1} + n_{z+1} \alpha_{z+1}) - n_e n_z (q_z + \alpha_z).
\]

(2.63)

\( \alpha_z \) is the total (radiative & dielectronic) recombination rate coefficient, and \( q_z \) is the ionization rate coefficient.

The timescales for ionization and recombination in the corona are of the order of tens to hundreds of seconds, compared to around \( 10^{-3} \) s for collisional excitation and deexcitation (Table 2.2). Because ionization and recombination are proceeding so slowly, significant departures from an equilibrium population of the different ionization stages are possible. In those cases, the ionization rate equations have to be solved together with the magnetohydrodynamic equations.

Peter et al. (2006) compared ionization and recombination timescales calculated from their MHD model to the hydrodynamic timescale of their model and found that the dynamic timescale is the largest timescale. They conclude that ionization equilibrium might be used as a first step for a spectroscopic analysis of three-dimensional MHD coronal models.
For the equilibrium case the ionization rate equation (2.63) reduces to
\[ n_z q_z = n_{z+1} \alpha_{z+1}, \]  
(2.64)
where the electron density has dropped out of the set of equations. Therefore, the ionization balance only depends on temperature. This equation is complemented by the requirement that the sum of the ion number densities is the elemental number density
\[ n_{el} = \sum_z n_z. \]  
(2.65)

The ionization balance populations of each ion are known from the literature and the ratio \( n_{ion}/n_{el} \) is tabulated in atomic databases such as the CHIANTI database. Throughout this work the ionization equilibrium calculations of Mazzotta et al. (1998) will be used.

**Figure 2.5:** Left: Ionization fractions of Si II, Si IV, C II, C III, C IV, O IV, O V, O VI, Ne VIII and Mg X as a function of temperature. Right: Ionization fractions of each ionic species of oxygen (O I - O IX) as a function of temperature according to Mazzotta et al. (1998).

Figure 2.5 (left) shows the ionization fractions of ions used in this study, i.e. Si II, Si IV, C II, C III, C IV, O IV, O V, O VI, Ne VIII and Mg X, as a function of temperature. The roman numbers refer to the ionization state of the ion, e.g. II implies that the ion is single-ionized, III implies a two-times ionized state, etc. The temperature where the ionization fraction of the respective line reaches its maximum is indicated by a vertical line of the same color as the respective ion fraction. These values are also listed in Table 3.1 in the column labeled log \( T_{ion} \).

Relative concentrations of each ionic species of oxygen (O I - O IX) are plotted as a function of temperature in Fig. 2.5 (right). Nearly all the oxygen is neutral at the lowest temperatures. Collisional excitation will create more O II at increasing
temperatures, and O ii will finally be dominant at temperatures that no longer allow recombination back to O i. The competition between ionization to the next higher ionization state and recombination to the subjacent ionization state continues with ions of the next state of charge when moving to higher temperatures, until eventually fully ionized O ix only is present at temperatures above $10^7$ K.

The value of the maximum abundance of individual ions highly depends on the ionization energy of the next state. As a consequence, ions representing closed shells, such as He-like O vii, have broad maxima with peak abundances near unity. On the other hand, ions representing the last electron in a shell, such as H-like O viii, have maximum relative abundances that are significantly below unity, because their ionization energy is small. Li-like ions, such as O vi, show a high-temperature tail due to dielectronic recombination from He-like O vii that exists over a wide temperature range. Since a considerable amount of energy is required to excite the autoionization state of O vi, the abundance of O vii results in significant dielectronic recombination to O vi which produces the high-temperature tail.

2.3.2.4 Abundances

In order to calculate the flux of a given element’s emission line (Eq. 2.49), the element’s abundance relative to hydrogen must be known. It is well known that the chemical composition of the solar corona differs from that of the underlying photosphere. Elements with a first ionization potential (FIP) $\leq 10$ eV (e.g. Fe, Mg, Si, Ca) are observed to be enhanced relative to those with FIP $\geq 10$ eV (e.g. O, Ne, S) in the corona by factors of 3-10 as compared to the photosphere. The mechanism causing this phenomenon known as the FIP effect is not yet understood. The FIP effect is not a ubiquitous feature of late-type stellar coronae, as spectroscopic observations of the nearby F5 IV star Procyon have shown, where the effect is not present in the corona. Whether the prevailing stellar situation is that of the Sun or that of Procyon is of fundamental interest to the physics of stellar outer atmospheres. For this study the solar photospheric abundances of the CHIANTI database (Dere et al., 1997; Landi et al., 2006) will be used (Table 2.3). It is common to list the logarithmic relative abundances normalized to $n_H = 10^{12}$ particles per unit volume, i.e. $\log A_{el} = 12 + \log \left( \frac{n_{el}}{n_H} \right)$.

2.3.2.5 Calculation of Electron Number Density

The ratio hydrogen to electron number density ($n_H/n_e$) must also be determined in order to calculate the flux in a given emission line (Eq. 2.49). Hydrogen and helium dominate the composition of the solar atmosphere. The abundances of heavier elements are many orders of magnitude lower (Table 2.3), which makes their contribution negligible for many applications. For a fully ionized plasma the electron
number density can be approximated by

\[ n_e = n_H + 2n_{He}, \]  

(2.66)

where \( n_H \) and \( n_{He} \) are the number densities of neutral hydrogen and neutral helium. Using the helium abundance of Table 2.3, the helium number density is \( n_{He} = 0.085n_H \). The electron number density (Eq. 2.66) can be rewritten as \( n_e = 1.17n_H \) or

\[ \frac{n_H}{n_e} \approx 0.85. \]  

(2.67)

From \( \rho = mn = \mu m_p n \) (section 2.2.2.2) it follows that the number of particles per unit volume is

\[ n = \frac{\rho}{\mu m_p} \approx 1.6 \frac{\rho}{m_p}, \]  

(2.68)

assuming \( \mu \approx 0.6 \) for the mean atomic weight in the solar corona (see appendix C). The number density is related to the electron density through

\[ n = n_e + n_H + 3n_{He} = \left( n_H + 2n_{He} \right) + n_H + n_{He} \]

\[ = 2n_H + 3n_{He} = 1.92n_e. \]  

(2.69)

Therefore, the electron number density can be directly calculated from the mass density \( \rho \) according to

\[ n_e = 0.52n = 0.87 \frac{\rho}{m_p}. \]  

(2.70)
2.4 Spectral Synthesis of Optically Thin Extreme Ultraviolet Emission Lines

Powerful atomic physics software packages, such as CHIANTI \cite{Dere1997}, can be used for the determination of the radiation spectrum emitted by the heated plasma, and such software can be handled straightforward if the plasma is in a state of ionization equilibrium. The emissivity of the plasma in a defined spectral line is proportional to the product of electron density squared and contribution function (section 2.3.2). The contribution function depends on atomic physics parameters and is unique to each spectral line (Eq. 2.51).

This section summarizes the procedure for the calculation of the emission for a set of optically thin ultraviolet emission lines from the model data. The lines that will be considered in this study are introduced in section 2.4.1. In section 2.4.2 the origin and usage of the atomic data will be described. Sections 2.4.3 and 2.4.4 outline the calculation of emissivities and spectral line profiles. In the final section 2.4.5 equations for the calculation of spectral maps are presented.

2.4.1 Emission Lines Synthesized in this Study

Emission lines synthesized in this study are listed in Table 2.4 supplemented with their wavelengths, formation temperatures and atomic transitions. The line formation temperature of the respective line is defined to be the temperature where the maximum of the contribution function $G(T, n_e)$ is found (see section 3.2.2). This temperature is roughly comparable, but not identical to the temperature of the maximum ionization fraction. The lines listed in Table 2.4 have been selected for two reasons. First, they cover the temperature range in the transition region and low corona, i.e. from $T \approx 10^4 - 10^6$ K. Second, the lines are observable with spectrometers such as SUMER/SOHO \cite{Handy1999} and EIS/Hinode \cite{Culhane2007}, and they are major lines in the bandpasses of the future Atmospheric Imaging Assembly (AIA) for the Solar Dynamics Observatory (SDO). Therefore, these synthesized lines will allow a comparison with the observed properties of the line profiles. The lines are optically thin, and the atomic transitions are predominantly excited by electron collisions (see section 2.3.1).

2.4.2 The CHIANTI Atomic Database

For the spectral synthesis of emission lines atomic models for the elements silicon, carbon, oxygen, neon, magnesium and iron will be applied. The analysis is carried out with data from the CHIANTI package \cite{Landi2006,Dere1997}. The CHIANTI package offers the possibility to choose between different tables for
### Table 2.4: Wavelengths and line formation temperatures of emission lines synthesized in this study along with their atomic transitions.

<table>
<thead>
<tr>
<th>line</th>
<th>wavelength [Å]</th>
<th>log(T/[K])</th>
<th>transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si II</td>
<td>1533.4</td>
<td>4.44</td>
<td>$3s^23p^2P_{3/2} - 3s^24s^2S_{1/2}$</td>
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<tr>
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</tr>
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<td>5.40</td>
<td>$2s^21S_0 - 2s2p^1P_1$</td>
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<tr>
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<td>5.48</td>
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<td>6.51</td>
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</table>

The ionization rates and elemental abundances and select the one that is most appropriate. It also provides a set routines in order to handle these data. For the calculation of the number density (Eq. 2.45) ionization equilibrium is assumed, and the [Mazzotta et al. (1998)] ionization fractions are selected. The validity of ionization equilibrium for modeling synthetic spectra from three-dimensional MHD models is critical and has been investigated by [Peter et al. (2006)]. Their discussion of ionization and recombination times in comparison to the dynamic timescale, i.e. the time to cross the line formation temperature, demonstrated that the ionization and recombination times in the coronal and transition region plasma are mostly smaller than the typical hydrodynamic timescales. This indicates that the approximation of ionization equilibrium is a good approach. However, for future models the inclusion of non-equilibrium ionization is important. Since line ratios will not be considered, the results do not depend on the specific abundances used in this study. The photospheric elemental abundances of [Grevesse and Sauval (1998)] included in the CHIANTI package were chosen.
2.4.3 Calculation of Emissivities

The temperature resolution provided by the MHD calculations is not sufficient to calculate the line emissivities in the transition region. The MHD calculations have been performed using a non-equidistant grid in the vertical direction with a spacing of neighboring grid points down to 75 km, but still, the temperature resolution in these data is too low. Typical contribution function widths of transition region emission lines range between 0.2-0.3 in \( \log T \). In order to reach a temperature resolution below these widths, the MHD results are interpolated in the vertical direction. As a consequence, the vertical distance between grid points in the transition region is reduced down to 40 km and temperature gradients are reduced down to 0.1 in \( \log T \) (spatial average). All the procedures mentioned in the preceding sections as well as the results presented in this thesis are derived from spatially interpolated MHD data.

The individual terms of Eq. 2.48 are determined independently with data from the CHIANTI database (see section 2.4.2) and are then composed to calculate the emissivity of the line at each grid point according to Eq. 2.45.

2.4.4 Calculation of Emission Line Profiles

It was noted in section 2.3.2 that the volume emissivity (Eq. 2.44) of an optically thin plasma is

\[ \epsilon_{\nu} = h \nu j_i \cdot n_j \cdot A_{ji} \cdot \psi_{\nu}, \]

where \( \psi_{\nu} \) is the emission line profile normalized to unity when integrated over all frequencies. Doppler broadening is the most important process contributing to the line profile of optically thin extreme ultraviolet emission lines formed in the transition region and corona. Each atom has its own velocity, and therefore the observed frequency of any photon emitted is different. This effect causes a spread of the line emission in wavelength. When considering an atom with a line-of-sight velocity component \( \xi \), the change in frequency associated with \( \xi \) is

\[ \nu - \nu_0 = \frac{\nu_0 \xi}{c}, \]  \hspace{1cm} (2.71)

where \( \nu_0 \) is the frequency in the rest-frame of the atom. In local thermodynamic equilibrium the distribution functions are close to Maxwellian, and the number of atoms in the velocity interval \( (\xi', \xi' + d\xi') \) is

\[ dn(\xi) = n \left( \frac{m}{2\pi k_B T} \right)^{1/2} \exp \left[ -\frac{m \xi^2}{2k_B T} \right] d\xi, \]  \hspace{1cm} (2.72)
where \( m \) is the atomic mass. A Gaussian-shaped line profile function is obtained by integration of Eq. 2.72

\[
\psi_{\nu} = \frac{1}{\pi^{1/2} \Delta \nu_D} \exp \left[ -\frac{(\nu - \nu_0)^2}{\Delta \nu_D^2} \right],
\]  

(2.73)

where \( \Delta \nu_D \) is the Doppler width of the spectral line defined as

\[
\Delta \nu_D \equiv \frac{\nu_0}{c} \left( \frac{2k_B T m}{m} \right)^{1/2}.
\]  

(2.74)

### 2.4.5 Calculation of Spectral Maps from Emission Line Profiles

For the purpose of convenience, calculations of the spectral profiles (Eq. 2.73) are performed in units of velocity. The line profile \( I_v \) at each grid point is

\[
I_v = \frac{I_{\text{tot}}}{\sqrt{\pi} \omega_{\text{th}}} \exp \left[ -\frac{(v - v_{\text{los}})^2}{\omega_{\text{th}}^2} \right],
\]  

(2.75)

where the line profile is assumed to have the thermal width

\[
\omega_{\text{th}} = \left( \frac{2k_B T m}{m} \right)^{1/2},
\]  

(2.76)

and the total intensity \( I_{\text{tot}} \) is the emissivity at the respective grid point (Eq. 2.45). \( v_{\text{los}} \) is the line-of-sight component of the velocity. The total spectrum is obtained by integrating the line profile function along the line-of-sight, e.g. the \( z \)-direction

\[
I_{\text{synth}} = \int_{z=0}^{z_{\text{max}}} I_v \, dz.
\]  

(2.77)

Thus, two-dimensional maps of spectra are derived, as they would be obtained by raster scans with spectrometers such as SUMER. When averaging the line profiles horizontally before integrating along the line-of-sight, a single average spectrum is obtained for each line. These synthetic spectra are analyzed in a way similar to observed spectra.

As the noiseless synthetic data are very close to Gaussian distributions, it is sufficient to calculate the profile moments with respect to velocity in order to obtain the line intensities, line shifts and line widths. Application of A.2, A.3 and A.4 (see appendix A) to Eq. 2.77 results in the expressions for the line intensity, line shift and line width

\[
I_{\text{synth}} = \int_{-\infty}^{+\infty} I_{\text{synth}} \, dv,
\]  

(2.78)
2 Theory and Methodology

\[
v_{\text{synth}} = \frac{1}{I_{\text{synth}}} \int_{-\infty}^{+\infty} v I_{\text{synth}}^v dv,
\]

\[
w_{\text{synth}} = \left[ \frac{2}{I_{\text{synth}}} \int_{-\infty}^{+\infty} (v - v_{\text{synth}})^2 I_{\text{synth}}^v dv \right]^{1/2}.
\]

When analyzing observed line widths, the thermal width (Eq. 2.76) is usually subtracted in order to obtain the nonthermal line width

\[
\omega_{\text{nt}}^{\text{synth}} = \left( \omega_{\text{synth}}^2 - \frac{2k_B T}{m} \right)^{1/2},
\]

which is a quantity that characterizes unresolved plasma motions.
3 Results of a High-Resolution Time-Series

Using the boundary conditions of section 2.1 numerical simulations of the atmosphere above a small active region have been carried out (Bingert, 2009). Based on these data line profiles of a set of extreme ultraviolet emission lines are calculated and investigated with respect to observable quantities like intensities, Doppler shifts and line widths. These quantities give an insight on the structure of the transition region and corona and on the origin and nature of the dynamics that lead to the observed phenomena. The comparison of spectra synthesized from the model with characteristics of observed spectra will provide an indication of the validity of the model.

In this chapter the data of a 30 minute high-resolution time-series is analyzed. Synthetic spectra are calculated and their properties are compared to observed spectra. In section 3.1 we start with a discussion of the existing atmospheric conditions in the simulation domain, such as temperatures, densities, velocities, magnetic field strength and the heating rate. In the subsequent section (section 3.2) results of the spectral analysis are presented for a set of extreme ultraviolet emission lines. Section 3.2 includes a discussion of the emission in various spectral lines (section 3.2.1) and their contribution functions (section 3.2.2). Sections 3.2.3, 3.2.4 and 3.2.5 focus on the investigation of Doppler shifts, on the intensity and Doppler shift variability and on the non-thermal line widths. A differential emission measure analysis is performed in section 3.3 in order to test the model with respect to the basic structure of the atmosphere. The underlying cause of the Doppler shifts is investigated in the final section of this chapter (section 3.4).

3.1 Investigation of Atmospheric Parameters

3.1.1 Discussion of Temperatures, Densities, Velocities and Magnetic Field Strength in the Simulation Box

Fig. 3.1 shows the variation of temperature, density, vertical velocity and magnetic field strength as a function of height in the simulation box. Horizontal averages of the logarithmic temperature and density are shown in the top row. The values have
been time-averaged. The bars indicate the scatter of the logarithmic temperatures in the horizontal layer.

Figure 3.1: Top left: Horizontally averaged logarithmic temperatures as a function of height (z) in the simulation box. The bars correspond to the scatter of the horizontal averages. The values are temporal averages of the time-series. Top right: Same as top left, but for particle number densities. Bottom left: Horizontal averages of the vertical plasma velocities at each height in the box. Bottom right: Horizontally averaged absolute values of the magnetic field strength $B$ at timestep $t=0$ min of the simulation.

The temperatures and densities in the simulation box cover large ranges from the photosphere to the corona. Therefore, the logarithm of these parameters is plotted as a function of height. The values vary considerably across horizontal planes as indicated by the length of the bars. The temperature profile calculated from the model shows the well-known steep rise in the transition region and the almost flat temperature gradient in the corona (see Fig. 1.3), thus demonstrating the validity of the model in the regions examined. A maximum temperature of 1.26 MK is reached at a height of 24 Mm. The average density at that height is approximately $3 \times 10^{15}$ particles/m$^3$ or $3 \times 10^{-12}$ kg/m$^3$. The variation of vertical velocity with height (Fig. 3.1 lower left) shows that the scatter of the velocities is large in the horizontal planes, but the average velocity at each height is found to be negative, i.e. the flow is directed towards the solar surface. This observation will be further
discussed in section 3.4. The horizontally averaged magnetic field strength decreases exponentially with height in the atmosphere (Fig. 3.4, lower right) from 500 G in the photosphere to 10 G in the upper transition region and 1 G in the lower corona in accordance to observations (compare Table 2.1).

3.1.2 Discussion of the Plasma-β Term and the Heating Term

In Fig. 3.2 histograms of plasma-β (left) and of the current density squared (right) both as a function of height in the simulation box are displayed. There are large variations in the horizontal planes, but the height dependence of the plasma-β term and the heating term displays a uniform trend. The current density squared (heating) shows its largest values at low heights as a consequence of the stressing of the magnetic field in a high-β domain. Through the photosphere and chromosphere the heating declines fast by orders of magnitude. In the upper chromosphere, the magnetic field gradually becomes “nearly force-free”, and therefore the scale height of the $j^2$ distribution approaches that of $B^2$ (Fig. 3.2 right). In the high chromosphere and above, the magnetic field is nearly space-filling in the sense that the magnetic field rapidly expands in the transition region to form funnel-like structures (Gabriel, 1976). The ratio of plasma pressure and magnetic pressure (plasma-β, Eq. 2.18) is less than unity almost everywhere in the upper atmosphere (Fig. 3.2 left; Table 2.1). The magnetic field thus dominates the dynamics, and plasma flows are confined along the magnetic field (section 2.2.2.1).

![Figure 3.2: Left: Histogram of plasma-β as a function of height in the simulation box. Right: Histogram of the current density squared as a function of height in the simulation box. Dashed lines show the horizontal mean values, dotted lines indicate minimum and maximum values. Darker colors correspond to higher filling factors. The dashed-dotted line on the right represents the logarithmic magnetic field strength squared.](image)

Fig. 3.3 shows histograms of the distribution of heating rates $\eta\mu_0j^2$ as a function of height for the box (upper left), a volume above the central part of an active region (upper right) and a volume above the quiet Sun (lower left). The regions are
Results of a High-Resolution Time-Series indicated by green, blue and red squares in Fig. 3.4. Although there is a relatively large scatter in the range of values at each height, the horizontally averaged heating rates show a smooth height dependence for these three volumes and an exponential decrease in the transition region and corona.

The same characteristics of heating have been proposed by Schrijver et al. (1999) and later been deduced with data from TRACE by Aschwanden et al. (2001). Further studies of heating models have supported these heating function characteristics, e.g. Gudiksen and Nordlund (2005a). It should be noted though that there is a dip in the heating function at a height of approximately 8 Mm that needs further consideration.

![Figure 3.3](image)

**Figure 3.3:** Histograms of the heating rates in the box (upper left), above an active region (upper right) and above the quiet Sun (lower left) as a function of height in the simulation box for t=0 min. The regions are indicated in Fig. 3.4. Dashed line shows the average heating rate, dotted lines indicate minimum and maximum values. Darker colors correspond to higher filling factors. Lower right: Average heating rate in the box (solid line), the active region (dashed line) and the quiet Sun (dotted line) in a single plot. The dashed dotted line indicates the average magnetic field strength squared at each height.

It is the active region where the heating rate has its best fit compared to the exponential decrease of the magnetic field strength squared with height (Fig. 3.3).
3.1 Investigation of Atmospheric Parameters

Figure 3.4: Vertical magnetic field strength at height zero when viewing the simulation box from above. Colored boxes indicate entire box (green), active region (red) and quiet Sun (blue) areas (Fig. 3.3). Dashed lines indicate cuts in x- and y-direction (Fig. 3.5).

lower right). In this region the heating rate decays exponentially between 10 Mm and 20 Mm, whereas the heating rates in the quiet Sun and in the entire box only start to decrease exponentially for heights beyond 20 Mm. A linear fit is applied to the active region logarithmic heating rate in the range 10 to 20 Mm in order to deduce a heating scale height $H$. The linear fit function derived is

$$\log_{10}(\eta \mu_0 j^2) = -4.04 - 0.11z,$$

with $z$ being the height of the box, and it is shown as a thick solid line in Fig. 3.3 (lower right) together with functions of the average heating rates in the the active region, the quiet Sun and the entire box. The heating scale height in the active region is found to be

$$H = \frac{1}{\ln 10 \cdot 0.11} \approx 4.1 \text{ Mm},$$

which is similar to the scale height of approximately 5 Mm in previous simulations by Gudiksen and Nordlund (2005a).

Examples of the current density squared $j^2$ as obtained by integration along the x- and y-direction of the box are presented in Fig. 3.5. The plots are scaled in a way that at each height after integration the image is divided by the average horizontal value. The top row shows integrated $j^2$ in the entire box, the middle row shows the integrated $j^2$ above the active region, and the bottom row shows integrated $j^2$ above the quiet Sun. These regions are indicated by dashed lines in Fig. 3.4. Bright regions in Fig. 3.5 indicate sites of enhanced heating, e.g. above the active region areas at x=40 Mm and y=15 Mm in the active region $j^2$ plot (Fig. 3.5 middle).
Regions of enhanced heating are also observed above the quiet Sun, e.g. at x=35 Mm and y=30 Mm (Fig. 3.5 bottom).

**Figure 3.5:** Maps of the current density squared $j^2$ integrated along the x-direction (left) and the y-direction (right). The plots are scaled in a way that at each height after integration the image is divided by the average horizontal value. The scaling is linear ranging from 0 (black, no heating) to 0.015 (white, heating). Top: for entire box, middle: above active region (AR), bottom: above quiet Sun (QS) according to the regions indicated in Fig. 3.4.
3.2 Spectral Analysis of Synthetic Spectra

3.2.1 Emissivities of Extreme Ultraviolet Emission Lines

The emissivities of a number of extreme ultraviolet emission lines (Table 2.4) are determined at each grid point in the simulation box by employing the procedure outlined in section 2.4.3. The lines have been selected for two reasons. First, the formation temperatures of these lines cover the range of temperatures in the transition region and low corona, i.e. \( T \approx 10^4 - 10^6 \) K. Second, the lines are observable with spectrometers such as SUMER/SOHO and EIS/Hinode and are major lines in the bandpasses of the future Atmospheric Imaging Assembly (AIA) of the Solar Dynamics Observatory (SDO). In Fig. 3.6 the emissivities of C IV (1548 Å) and Ne VIII (770 Å) are plotted after having been integrated along the y-, x- and z-direction at the start of the simulation time \( (t=0 \, \text{min}) \) and at \( t=15 \, \text{min} \). The starting point was chosen to be in the middle of the real simulation time \( (\approx 30 \, \text{min}) \) in order to ensure that the boundary conditions that were present at the beginning of the simulation will be relaxed by the start of the time-series.

The appearance of the synthesized corona in the lower transition region line C IV (1548 Å) (line formation temperature \( T_f = 10^5 \) K) as shown in the top row of Fig. 3.6 is remarkably different from that in Ne VIII (770 Å) \( (T_f = 6.3 \times 10^5 \) K) in the low corona shown in the bottom row of Fig. 3.6. The C IV images in Fig. 3.6 reveal the presence of low-lying cool, dense loops and the enhancement of the transition region emission in the footpoint regions of hot loops. However, the majority of the emission of typical transition region lines is emitted in cooler structures supporting earlier ideas of the structure of the low corona (Dowdy et al., 1986). Rather, structures are washed out and less sharp at high temperatures in the Ne VIII images, reflecting the efficient heat conduction at such temperatures. These findings are in good agreement with observations. When comparing intensity maps of two timesteps within a 15 minute period of time (Fig. 3.6), changes in the transition region structure in the C IV (1548 Å) line are visible, while the coronal appearance in the Ne VIII (770 Å) line is pretty much the same in terms of intensity. The transition region directly corresponds to the changes in the photospheric magnetic structure, while the corona is less sensitive to such changes.

Fig. 3.7 gives an impression of the synthesized emission of the Ne VIII (770 Å) line when looking at the simulation box from different angles at \( t=0 \, \text{min} \) simulation time. When rotating the simulation box around the z-axis, i.e. for a different line-of-sight, loop-like structures appear and “dissolve” again.
Figure 3.6: Top row: C IV (1548 Å) emissivities integrated along the y-, x- and z-axis in the simulation box for timestep $t=0$ min. The emission is plotted on a logarithmic scale that corresponds to 0.1-10 times the average emission of the reference timestep ($t=0$ min). Second row: Same as top row, but for timestep $t=15$ min. Third row: Ne VIII (770 Å) emissivities integrated along the y-, x- and z-axis in the simulation box for timestep $t=0$ min. Bottom row: Same as third row, but for $t=15$ min. The transition region line C IV (1548 Å) has a line formation temperature of $T = 10^5$ K, the coronal line Ne VIII (770 Å) is formed at $T = 6.3 \times 10^5$ K.
3.2 Spectral Analysis of Synthetic Spectra

Figure 3.7: Synthesized emission of the Ne\textsuperscript{viii} (770 Å) line when looking at the simulation box from different angles at t=0 min simulation time. The different colors indicate the strength of the emission (blue: emissivity $\epsilon > 10^{-5}$ W/m$^3$, green: $\epsilon \approx 10^{-6}$ W/m$^3$, red: $\epsilon \approx 10^{-7}$ W/m$^3$, white: no emission). On the bottom of the box the vertical magnetic field is shown scaled from -1000 G (black; inward directed field) to +1000 G (white; outward directed field). \textit{Top:} Loop-like structures appear in the Ne\textsuperscript{viii} emission. \textit{Bottom:} The structures "dissolve" when the simulation box is rotated around the z-axis, i.e. for a different line-of-sight. Images produced with VAPOR (Clyne and Rast, 2003; Clyne et al., 2007).
3 Results of a High-Resolution Time-Series

3.2.2 Contribution Functions and Line Formation Temperatures

The contribution of selected lines to the total emission of the line as a function of temperature can be calculated by integrating the emissivities of the respective line in the simulation box in small log\(T\) intervals. In Fig. 3.8 such contribution functions are presented for the Si\textsc{iv} (1394 Å), C\textsc{ii} (1335 Å), C\textsc{iii} (977 Å) and C\textsc{iv} (1548 Å) lines. The log\(T\) interval for the integration is 0.01, and the time-averaged contribution functions have been normalized. The lines that are formed between log\(T=4.7\) and log\(T=6.2\) are roughly Gaussian-shaped. The Gaussian fits are shown as solid lines in Fig. 3.8 and the fit values for the center and the full-width-at-half-maximum of the distribution are listed in Tab. 3.1 (labeled log\(T_c\) and FWHM log\(T_c\)). The contribution function of lines formed at low temperatures, i.e. Si\textsc{ii} (log\(T_f=4.4\)) and C\textsc{ii} (log\(T_f=4.7\)) show deviations from the Gaussian shape that have been discussed by Peter et al. (2006). For the Gaussian fit of the Si\textsc{ii} contribution function only temperatures down to log\(T=4.3\) are taken into account. The reason for this is that the observed emission, proportional to the density squared, strongly increases for lower temperatures due to the very high density of the solar atmosphere at such temperatures. The mean values of the Gaussian fits to the contribution functions log\(T_c\) and the line formation temperatures log\(T_f\) match well as can be seen from Tab. 3.1.

3.2.3 Investigation of Doppler Shift Maps

Two-dimensional Doppler shift maps are derived from the emission line profiles of the various lines according to the procedure outlined in section 2.4.5. First, starting with the emissivity, temperature and velocity at each grid point, a line profile is calculated according to Eq. 2.75. Following this, an integration along the line of sight, i.e. the vertical direction, is performed and the line profile moments are calculated at each spatial location. Doppler shift maps of C\textsc{ii} (1335 Å), C\textsc{iv} (1548 Å), O\textsc{vi} (1032 Å) and Ne\textsc{viii} (770 Å) are shown in Fig. 3.9 for the first timestep \(t=0\) min of the simulation. These synthesized spectral maps can be analyzed the way spatial scans are that are being obtained by the SUMER instrument onboard SOHO (Peter and Judge, 1999).

The Doppler shift images of both the C\textsc{ii} (1335 Å) and C\textsc{iv} (1548 Å) line formed in the low and middle transition region (\(T_f ≈ 4 \times 10^4\) K and \(T_f ≈ 10^5\) K) show a lot of small-scale structure and dynamics that indicate the direct response of the transition region to the changes in the photospheric magnetic structure. As can be seen in Fig. 3.10 Doppler shifts in the transition region lines show significant changes within a few minutes.
### 3.2 Spectral Analysis of Synthetic Spectra

<table>
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<th>( \lambda ) [Å]</th>
<th>( \log(T_{\text{ion}}) ) [K]</th>
<th>( \log(T_f) ) [K]</th>
<th>( \log(T_c) ) [K]</th>
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<td>0.30</td>
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<td>O VI</td>
<td>1031.9</td>
<td>5.48</td>
<td>5.48</td>
<td>5.50</td>
<td>0.23</td>
</tr>
<tr>
<td>Ne VIII</td>
<td>770.4</td>
<td>5.80</td>
<td>5.80</td>
<td>5.89</td>
<td>0.25</td>
</tr>
<tr>
<td>Mg X</td>
<td>624.9</td>
<td>6.06</td>
<td>6.05</td>
<td>6.08</td>
<td>0.36</td>
</tr>
<tr>
<td>Mg VIII</td>
<td>609.8</td>
<td>6.06</td>
<td>6.05</td>
<td>6.08</td>
<td>0.12</td>
</tr>
<tr>
<td>Fe X</td>
<td>185.2</td>
<td>5.56</td>
<td>5.63</td>
<td>5.73</td>
<td>0.40</td>
</tr>
<tr>
<td>Fe XI</td>
<td>184.5</td>
<td>5.99</td>
<td>5.99</td>
<td>6.05</td>
<td>0.17</td>
</tr>
<tr>
<td>Fe XII</td>
<td>188.2</td>
<td>6.07</td>
<td>6.07</td>
<td>6.09</td>
<td>0.12</td>
</tr>
<tr>
<td>Fe XIII</td>
<td>195.1</td>
<td>6.13</td>
<td>6.13</td>
<td>6.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Fe XIV</td>
<td>203.8</td>
<td>6.20</td>
<td>6.20</td>
<td>6.11</td>
<td>0.07</td>
</tr>
<tr>
<td>Fe XV</td>
<td>264.8</td>
<td>6.27</td>
<td>6.27</td>
<td>6.13</td>
<td>0.05</td>
</tr>
<tr>
<td>Fe XVI</td>
<td>284.2</td>
<td>6.32</td>
<td>6.32</td>
<td>6.13</td>
<td>0.04</td>
</tr>
<tr>
<td>Fe XVII</td>
<td>263.0</td>
<td>6.42</td>
<td>6.41</td>
<td>(6.11)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Ca XIV</td>
<td>193.9</td>
<td>6.51</td>
<td>6.51</td>
<td>(6.11)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

*a For the Gaussian fit of the Si ii contribution function only temperatures down to \( \log T = 4.3 \) are taken into account.

*b Due to the low number of particles in the simulation box above \( \log T = 6.1 \), the temperatures derived from the contribution functions of the Fe xvi and Ca xiv lines are not reasonable.

**Table 3.1:** Line formation temperatures according to different definitions: \( \log T_{\text{ion}} \) is the temperature of the maximum ionization fraction based on the ionization equilibrium calculations of Mazzotta et al. (1998) (see section 2.3.2.3), \( \log T_f \) is the maximum of the contribution function \( G(T) = \frac{\nu_{\text{ion}}}{\nu_{\text{el}}} T^{-1/2} \exp \left( -\frac{h\nu}{kT} \right) \) (section 2.3.2.2), and \( \log T_c \) corresponds to the mean values of the Gaussian fits to the line contribution functions (section 3.2.2). The last column shows the corresponding full-width-at-half-maximum values of the Gaussian fits to the line contribution functions.

The coronal lines O vi (1032 Å) and Ne viii (770 Å) display their dynamic structure in the Doppler maps (Fig. 3.11). While in the corona the heat conduction quickly redistributes the deposited energy resulting in a smooth appearance of the intensity maps that does not change a lot in time, the dynamic response to the heating is still present showing up as flows causing the Doppler shifts. Like in the transition region the coronal Doppler shifts (Fig. 3.11) show some variation on small spatial scales.
Figure 3.8: Normalized emissivity contribution functions (+) of the Si IV (1394 Å), C II (1335 Å), C III (977 Å) and C IV (1548 Å) lines. The log \( T \) interval for the integration of the emissivities is 0.01. The solid lines show Gaussian fits to the data. The fit values for both the center of the distribution and the full-width-at-half-maximum are listed in Tab. 3.1 (labeled log \( T_c \) and FWHM(log \( T_c \))). The values presented are temporal averages of the entire time-series.

within minutes corresponding to the timescale of the photospheric driver.

### 3.2.3.1 The Spread of Doppler Shifts

Histograms of the distributions of Doppler line shifts of the model are presented in Fig. 3.12. They can be compared to the Doppler shifts obtained from the analysis of transition region emission line profiles observed with SUMER (Fig. 3.13; Peter, 1999). The C IV (1548 Å) Doppler shift distribution of both the simulation and the observation peak at velocities of 3-4 km/s. The distribution of the modeled C IV (1548 Å) Doppler shifts is only slightly asymmetric, but lacks the contribution of high redshifts above 10 km/s when compared to the observations. This results in
3.2 Spectral Analysis of Synthetic Spectra

![Doppler maps of various spectra](image)

**Figure 3.9:** Doppler maps of C\textsc{ii} (1335 Å), C\textsc{iv} (1548 Å), O\textsc{vi} (1032 Å) and Ne\textsc{viii} (770 Å) at t=0 min. The C\textsc{ii} and C\textsc{iv} maps are scaled from -10 km/s (blueshifts) to +10 km/s (redshifts); the O\textsc{vi} and Ne\textsc{viii} maps are scaled from -15 km/s to 15 km/s and from -20 km/s to 20 km/s.

A lower median value for the C\textsc{iv} (1548 Å) Doppler shifts of the simulation (3.6 km/s) compared to the observations (5 km/s). Similar full-width-at-half-maximum (FWHM) values of approximately 10 km/s are obtained for both C\textsc{iv} (1548 Å) and Ne\textsc{viii} (770 Å). Peter (1999) observed a larger FWHM for the C\textsc{iv} (1548 Å) Doppler shift distribution (16 km/s) and about the same FWHM for the Ne\textsc{viii} (770 Å) Doppler shifts (10 km/s). The small fraction of blueshifts in the asymmetric Ne\textsc{viii} (770 Å) distribution of the model causes a large discrepancy between the median values of Doppler shift in the model (8 km/s) and in the observation (-3 km/s).
3 Results of a High-Resolution Time-Series

Figure 3.10: 30 minute time-series of C\textsc{iv} (1548 Å) Doppler shift maps. The images are scaled from -10 km/s (blueshifts) to +10 km/s (redshifts). The corresponding C\textsc{iv} (1548 Å) Doppler shift map at t=0 min is shown in Fig. 3.9 (upper right).

3.2.3.2 Comparison of Doppler Shifts from Average Spectrum and Mean Doppler Shift of All Spectra

Whereas spatial maps of Doppler shifts were discussed only in the previous section, average shifts of spectra will now be considered, in addition. The average shift of the spectrum is obtained by averaging the line profiles of all spatial pixels in the horizontal plane before calculating the profile moments. The shift of this average spectrum for each line can then be compared to the corresponding average Doppler shift of the two-dimensional maps.
3.2 Spectral Analysis of Synthetic Spectra

Figure 3.11: 30 minute time-series of O\textsc{vi} (1032 Å) Doppler shift maps. Images are scaled from -15 km/s (blueshifts) to +15 km/s (redshifts). The corresponding O\textsc{vi} Doppler shift map at t=0 min is shown in Fig. 3.9 (lower left).

Fig. 3.14 (top) shows the shifts of the average spectra as a function of line formation temperature for a set of transition region and coronal lines at two different timesteps of the simulation 10 minutes apart. The match between the Doppler shifts in the observation and the ones of the 3D MHD model is remarkable in the transition region up to line formation temperatures of log $T$=5.5. The difference between modeled and observed Doppler shift is less than 1σ for most lines in the transition region. However, for the coronal lines of Ne\textsc{viii} (770 Å) and Mg\textsc{x} (625 Å) the
observed blueshifts are not reproduced by the MHD model. This is a well-known problem that could be due to the boundary conditions applied (see section 5).

The bottom panel of Fig. 3.14 shows the comparison between the line shifts of the average spectrum and the mean line shift of all spectra at t=5 min. For most lines the Doppler shifts of the average spectrum are clearly larger than the mean.
3.2 Spectral Analysis of Synthetic Spectra

Doppler shifts of all spectra. The Doppler shifts of the average spectra and the mean spectra are comparable only for lines in the low transition region (Si ii (1533 Å) and C ii (1335 Å)) and in the corona (Ne viii (770 Å) and Mg x (625 Å)). This observations suggest an examination of the relationship between Doppler shifts and the resolution of the spectral map (Fig. 3.15). Starting with 256x256 grid points, spectral maps of a sequence of 128x128, 64x64, 32x32, 16x16, 8x8, 4x4, and 2x2 grid points are calculated thus averaging the line profiles of an increasing number of grid points. Following this, average spectrum Doppler shifts and mean Doppler shifts are calculated as outlined above.

In Fig. 3.15 Doppler shifts of spectra with different resolution are presented as a function of line formation temperature for the timestep t=30 min. The number of grid points of the spectral map from which the Doppler shifts are calculated is shown by different symbols (see legend). The 1x1 example corresponds to the shift of the average spectrum and the 256x256 example is identical to the mean Doppler shift of all spectra. A decrease in Doppler shifts is observed for higher spatial resolution spectral maps. The dependence of the Doppler shifts on the resolution of the spectral map is most obvious at transition region temperatures.

3.2.4 Intensity and Doppler Shift Variability

SUMER/SOHO observations of EUV emission lines have shown that transition region lines are redshifted on average, whereas coronal lines are blueshifted (Peter and Judge, 1999). Models proposed to explain the observed line shifts include siphon flows through loops, explosive events, waves due to nano-flares or return of spicular material. Brkovic et al. (2003) studied SUMER and CDS time-series of EUV spectra of the quiet-Sun region at disk center and found a high correlation between average net Doppler shifts and relative brightness variabilities of the emission lines. They suggest a connection between these two quantities.

A positive correlation between network emission and redshifts in transition region lines, as found by e.g. Hansteen et al. (2000), is compatible with the model of nano-flares occurring at the top of coronal loops. The nanoflares can generate MHD waves that propagate downward along the magnetic field lines towards the loop footpoints that are anchored in the network (Hansteen, 1993). The net redshifts of transition region lines are a result of the correlation between the intensity and velocity that occurs in downward propagating acoustic waves.

We follow the approach of Brkovic et al. (2003) and calculate the average intensity variability and average Doppler shift variability of the spectral maps for the duration of the time-series in order to investigate a possible correlation between both quantities. The time variability of the intensity $\delta I$ is described by the root-mean-square
Figure 3.14: Comparison of observed Doppler shifts of transition region and coronal lines (dashed line, Peter and Judge (1999)) with the predictions of the 3D MHD model. **Top:** Doppler shifts of the average spectrum for timesteps $t=5$ min (black diamonds) and $t=15$ min (gray diamonds). The height of the bars/rectangles corresponds to the standard deviation of the mean value of the respective Doppler shift map. **Bottom:** Doppler shifts of the average spectrum (black diamonds) and mean Doppler shift of all spectra (gray diamonds) at timestep $t=5$ min.

(RMS) variation of the intensity during the time-series,

$$
\delta I = \sqrt{\frac{1}{N} \sum_{N} (I_N - \langle I \rangle)^2},
$$

where $N$ is the number of timesteps of the time-series, $I_N$ is the intensity at the
3.2 Spectral Analysis of Synthetic Spectra

Figure 3.15: Doppler shifts as a function of line formation temperature at $t=30$ min for spectral maps with different resolution. The numbers in the upper left of the figure correspond to the number of gridpoints of the spectral map: 256x256 corresponds to the original spectral map, 128x128 corresponds to the map where 2x2 columns are combined in a single column, 64x64 corresponds to the map where 4x4 columns are combined in a single column, etc. The Doppler shifts of the 1x1 map corresponds to those of the average spectrum (see Fig. 3.14).

respectively, and $\langle I \rangle$ is the average intensity of the entire duration of the simulation. The relative variability $\delta I/\langle I \rangle$ is defined as the ratio of RMS intensity variation and average intensity. The parameters $\langle I \rangle$, $\delta I$ and $\delta I/\langle I \rangle$ are determined for each spatial gridpoint for the respective line. Finally, spatial averages over all grid points are calculated for each line. The same procedure is applied to determine the RMS fluctuations of the (relative) Doppler shifts,

$$\delta v = \sqrt{\frac{1}{N} \sum_{N} (v_N - \langle v \rangle)^2},$$

(3.4)

where $\langle v \rangle$ is the average Doppler velocity of the time-series, and $v_N$ is the Doppler velocity at the respective timestep.

Fig. 3.16 shows the relative intensity variability (panel a), the Doppler shift variability (panel b), and the mean Doppler shift (panel c) as a function of line formation temperature, as well as the Doppler shift versus relative intensity variability (panel d). The error bars correspond to the root-mean-square values when spatially
averaging over all grid points. The thick dashed lines indicate the trend of the observed quantities as found for the quiet Sun (observed data in Fig. 3.16 a and b from [Brkovic et al. 2003], observed data in Fig. 3.16 c from [Peter and Judge (1999)]). The dashed line in panel d indicates a linear fit between the relative intensity variability and the Doppler shift as found in [Brkovic et al. 2003]. As discussed in [Zacharias et al. (2009a)], the relative intensity variability (Fig. 3.16 a) shows a maximum at lower line formation temperature than the observed quiet Sun data. The root-mean-square values of Doppler shift show an increase with line formation temperature (Fig. 3.16 b) that is less pronounced than the increase found for observations. The Doppler shifts of the simulation data increase with line formation temperature up to 7 km/s at $T \approx 10^6$ K. On the quiet Sun, observed redshifts increase up to 10 km/s at log $T = 5.2$.

**Figure 3.16:** a : Relative intensity variability of spectral maps of transition region and coronal lines as a function of line formation temperature b : Doppler shift variability as a function of line formation temperature. c : Mean Doppler shift as a function of line formation temperature. d : Mean Doppler shift versus relative intensity variability.
Active stars with convective envelopes show a complicated surface magnetic field distribution. Magnetic flux densities can reach values up to 100 times larger than those observed for the Sun and imply an increased heating of the upper layers of the stars’ atmospheres. The trend of redshift with line formation temperature of late-type dwarf stars, such as Procyon, AB Dor, α Cen A, α Cen B, ε Eri, AU Mic, has been investigated in many works (e.g. Wood et al., 1997; Refield et al., 2002; Sim and Jordan, 2002). For most of these stars an increase in velocity offset with increasing line formation temperature is observed and sooner or later a decrease with increasing temperature, the same general behavior as for solar line shifts (Peter and Judge, 1999). The setup of the model used to investigate properties of the synthesized emission lines applies a higher magnetic flux density at photospheric levels than observed for the quiet Sun. The models can therefore not be considered as reproducing the solar quiet Sun emission, but rather as a step towards more complex surface magnetic field topologies, and thus as a step towards investigating the coronae of stars more active than our Sun. Our findings indicate that stars with higher magnetic activity than the quiet Sun show a maximum redshift at higher temperatures than the quiet Sun.

3.2.5 Non-Thermal Line Widths

Unresolved motions, either due to a mass flow or waves, result in a broadening of the line. It is well known from previous EUV and far-ultraviolet (FUV) measurements of solar coronal and transition region lines that spectral emission line profiles are considerably broader than expected on the basis of the thermal motions of the emitting ions (Chae et al., 1998). The implied velocities are generally in the range of 5-30 km/s for temperatures in the range 0.02-1 MK. Chae et al. (1998) and Peter (2001) have determined nonthermal velocities in the quiet Sun at temperatures between $10^4$ K and $2 \times 10^6$ K by measuring the widths of a number of EUV and FUV lines observed with SUMER/SOHO. They found that the nonthermal velocity at temperatures below $2 \times 10^4$ K is smaller than 10 km/s. The velocity increases with temperature, reaches a peak value of 30 km/s around $T = 3 \times 10^5$ K and then decreases again with temperature. There are a number of possible causes of the broadening, including turbulence and unresolved wave motions, nanoflares, or fine-scale laminar flows, each of which may be important for the energy transport and heating of the corona.

Fig. 3.17 shows the non-thermal line widths of the spectra as a function of line formation temperature for emission lines in the temperature range $T = 3 \times 10^4$ - $1.3 \times 10^6$ K. The observed trend is indicated by the thick dashed line. Except for the coronal lines, the line widths of the average spectra are of the order of 10 km/s for most lines investigated (Fig. 3.17). An increase in non-thermal line width is observed for both the O VI (1032 Å) and the Ne VIII (770 Å) lines. The Mg X (625
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Å line shows a non-thermal width of approximately 10 km/s again. The mismatch between observation and model line widths is strongest in the transition region.

A recent study by [Peter et al. (2006)] showed that in the corona and the low transition region the nonthermal widths from synthesized spectra match roughly the observed values, but in the middle transition region the synthetic spectra show significantly smaller widths than the observations. The authors argue that one possible explanation for this observation might be that the high nonthermal broadening observed on the Sun is a consequence of the small-scale velocities connected with the heating process itself, i.e., the nanoflares induced by the flux braiding. Because of their tiny spatial scale, these nanoflares cannot be modeled in the MHD simulation. This effect is expected to have the largest effect in the middle transition region, where the timescales are shortest. However, models with increased spatial resolution are necessary to show if this mismatch with the observations can be resolved.

![Figure 3.17: Comparison of observed non-thermal line widths (thick dashed line, data from Peter, 2001 and Chae et al., 1998) with non-thermal line widths calculated from spectra derived from the 3D MHD model. Line widths of the average spectrum at t=5 min are shown in black, line widths of the average spectrum at t=15 min are shown in gray. The height of the error bars/rectangles corresponds to the standard deviation of the mean value of the respective line width map.](image-url)
3.3 Differential Emission Measure Analysis

The differential emission measure distribution (DEM) is a physical quantity related to the electron density and to the temperature gradient of a plasma (Eq. 2.52) and can be directly obtained from the MHD model. The DEM function can be also obtained by inversion of Eq. 2.53 using the flux measured in a spectral line (Eq. 2.59) and the temperature-dependent contribution function (Eq. 2.51) that confines the emission measure to a limited temperature range.

In this section the DEM function derived from observations will be compared to the DEM functions determined by direct calculation or inversion from synthesized emission of the MHD model. The DEM inversion method is outlined and applied to the synthetic data of the model in section 3.3.1. Quiet Sun DEM observations are described in section 3.3.2. In section 3.3.3 results of the DEM derived directly from the MHD model parameters are presented.

3.3.1 Differential Emission Measure Inversion with the Atomic Database CHIANTI

The function DEM(T) can be derived by inversion of Eq. 2.53, which is a Fredholm integral equation of the first kind. Such inversion problems are mathematically ill-posed. Small changes in the emission line flux result in large changes of DEM(T). On the other hand, a large range of DEMs is capable of producing the spectrum of observed fluxes within predefined arbitrarily small errors.

The SolarSoft IDL routine chianti_dem.pro is applied to perform a differential emission measure analysis of the EUV spectra using the CHIANTI atomic database. The fitting method consists of an iterative procedure in which the DEM(T) is described by a cubic spline function through a number of mesh points. This procedure is outlined in Landi and Landini (1997) and in Landi and Young (2009). The DEM values are varied until the best fit with the measurements is achieved. The smoothness of the inversion result is controlled through the number of mesh points. The best DEM fits are obtained when spectral lines are selected that are not density-sensitive and that cover a wide range in formation temperature. The inversion results are influenced by various parameters, e.g. the number and position of the mesh points of the spline that represents the DEM. Also, the derivation of the DEM function is sensitive to noise and errors in spectral and atomic data (see e.g. McIntosh et al., 1998). For the calculation of the line intensities (section 3.2.1) and the computation of the DEM curve, all atomic data are taken from CHIANTI version 5.2 (Landi et al., 2006). The DEM curve (Fig. 3.13) has been derived adopting the ion abundance of Mazzotta et al. (1998) and the standard element abundances of Grevesse and Sauval (1998).
3 Results of a High-Resolution Time-Series

The silicon, carbon, oxygen, neon and magnesium emission lines listed in the upper half of Table 3.1 are being used for the inversion. The lines are not density-sensitive, and they are emitted by various elements over a large temperature range. The diamonds in Fig. 3.18 represent the DEM values and the solid line shows the inversion fit. The bars indicate the scatter of the DEM values for a given flux uncertainty of 30% (Eq. 2.53).

![Figure 3.18: Comparison of the DEM distributions for the model and for the observations. The dashed-dotted line shows the DEM directly derived from the MHD model, solid and dashed lines show the results of a DEM inversion applied to the model data and to observed quiet Sun emission line fluxes (Wilhelm et al., 1998).](image)

3.3.2 The Differential Emission Measure of the Quiet Sun

In addition to the results presented in section 3.3.1, Fig. 3.18 also shows results obtained from a DEM inversion of emission lines observed with SUMER/SOHO (Wilhelm et al., 1998; thick dashed line). The authors observed quiet Sun emission line fluxes whereas our model setup represents a small active region. Therefore, in order to be comparable it was suggested by Peter et al. (2004) to increase the observed values by a factor two. This allows to focus on the shape of the DEM distributions. The quiet Sun DEM distribution as shown in Fig. 3.18 can be characterized by some general features, including a minimum near $T = 3 \times 10^5$ K, a power-law distribution with negative slope at temperatures below $T = 3 \times 10^5$ K, and a power-law distribution with positive slope between $T = 3 \times 10^5$ K and $T = 10^6$ K. For temperatures exceeding $T = 10^6$ K, the observed DEM values decrease again. As can be seen from
3.3 Differential Emission Measure Analysis

Fig. 3.18 (solid line), the DEM of the model derived by inversion (section 3.3.1) also shows an increase at temperatures below $T = 10^5$ K.

3.3.3 Differential Emission Measure Distribution Derived from the MHD Model

In addition, the DEM can be directly derived from the MHD model according to Eq. 2.52. The differential emission measure integrated over temperature intervals of 0.1 in $\log T$ is the third function shown in Fig. 3.18 (dash-dotted line). The displayed values are temporal averages over all timesteps of the simulation. The overall shape of the DEM is similar to the observed DEM distribution (section 3.3.2) showing small changes only in the course of the simulation. In particular, the rise of the DEM distribution at temperatures below $T = 10^5$ K is pictured by the model. The distribution also has a local minimum near $\log T = 5.2$ and shows an inflection point at about one million Kelvin. This impressive congruence has been accomplished without further fine-tuning of the model.
3 Results of a High-Resolution Time-Series

3.4 Mass Balance in the Corona

After the model’s conformity to observations has been shown in the previous sections demonstrating adequate validity, the model will now be applied to parameters like the coronal mass and mass fluxes in order to understand the formation of line shifts that have already been discussed in section 3.2.3. This will open up options to find a better approach to the formation of blueshifts that are currently unsatisfactory generated by the model.

In the following the mass balance of the corona will be investigated (section 3.4.1). In section 3.4.2 mass flows in the transition region and corona are calculated, and velocities derived from Doppler shifts of the model will be compared to velocities derived from these mass fluxes. Whereas average temperature layers in the box have already been calculated and discussed in section 3.1.1 (compare Fig. 3.1), isosurfaces of constant temperature will also be considered in the following. These isosurfaces are supposed to be the source regions of lines the Doppler shift of which is being observed.

3.4.1 Variation of the Coronal Mass

Changes of the number of particles in the simulation box are expected to be associated with both redshifts (mass moving upwards) and blueshifts (mass moving downwards). The temporal variation of the number of particles is

\[ \delta = \frac{N - \langle N \rangle}{\langle N \rangle}, \tag{3.5} \]

where \( N \) is the total number of particles, and \( \langle N \rangle \) is the average total number of particles for the duration of the simulation, i.e. 30 minutes. Variations above the isosurfaces of \( \log T = 4.1, 4.5, 5.0, 5.5 \) and 6.0 are shown in Fig. 3.19. The isosurfaces vary for each timestep (left) or are kept constant in time at timestep \( t=0 \) min (right). The same variation is valid for the coronal mass because of the mass being proportional to the particle number.

A significant decrease in coronal mass is seen for the isosurface \( \log T=6.0 \) (Fig. 3.19 top left). The decrease occurs within first five minutes and afterwards the function is flat. All other isosurfaces show a permanent increase after approximately \( t=5 \) min. Looking at the isosurfaces constant in time (Fig. 3.19 top right) the function is mainly characterized by a decrease during the first minutes and variations are found to be smaller than those of the isosurfaces that vary at each timestep. In the low atmospheric layers of high density mass variations as observed in the transition region and corona are small compared to the total mass of the low layers (compare Table 2.1), and as a consequence mass variations cannot be observed.
3.4 Mass Balance in the Corona

Figure 3.19: Top left: Variation of the number of particles above isosurfaces of \( \log T = 4.1, 4.5, 5.0, 5.5 \) and 6.0 that are varying in time, i.e. a slightly different isosurface - that of the respective timestep - is considered at each timestep. Top right: Same as top left, but for the \( t=0 \) min isosurfaces fixed in time. Bottom left: Variation of the number of particles in volumes enclosed by \( \log T \) isosurface with a spacing of \( \log T = 0.5 \) kept fix in time. Bottom right: Same as bottom left, but for isosurfaces with a spacing of \( \log T = 0.2 \) kept fix in time. The temperature thresholds are indicated by different levels of gray (see legend).

In addition to the entire coronal mass, the mass enclosed by isosurfaces of different temperatures is investigated. The bottom row of Fig. 3.19 shows that the variations for isosurfaces with a spacing of 0.5 in \( \log T \) (left) and of 0.2 in \( \log T \) (right) are in the 2% range. A continuous mass decrease that would correspond to redshifts in the model is not found in any of the temperature intervals. This allows the conclusion that a global sinking of the atmosphere can be excluded as a mechanism causing the Doppler shifts of the model.

Based on the 2\% variation range found (Fig. 3.19), the vertical velocity that would be observed at \( T = 10^5 \) K can roughly be estimated to be \( v = 0.6 \) km/s. This number has been calculated according to \( v = \delta \cdot \frac{N}{A \cdot \tau} = \delta \cdot \frac{H}{A} \), where \( N = n \cdot A \cdot H \) is the total number of particles in the corona, \( n = 10^{15} \) m\(^{-3} \) is the particle density in the transition region, \( A \) is the horizontal area cut through the box, and \( \tau = 15 \)
min is the timescale of the variation. The value assumed for the scale height is $H = 25 \, \text{Mm}$, larger than the scale height in the transition region ($H \approx 5 \, \text{Mm}$ at $T = 10^5 \, \text{K}$), therefore $v = 0.6 \, \text{km/s}$ is an upper estimate. We can conclude that the vertical velocities in the box only are too small to explain the observed redshifts in the model.

### 3.4.2 Mass Fluxes in the Solar Transition Region and Corona

In this section the vertical velocity $v_z$ and the mass flux $\rho \cdot v_z$ through isosurface temperature layers are investigated (Fig. 3.20). Both quantities are found to be negative corresponding to downflows through the isosurface temperature layers. Higher up in the atmosphere, at coronal temperatures the vertical downflow velocities are found to be larger compared to those at transition region temperatures (Fig. 3.20, top left). Since the density decreases with height in the atmosphere, this trend is reversed for the particles flux (Fig. 3.20, top right), i.e. the highest particle fluxes pointing towards the solar surface are observed for isosurfaces of low temperature.

![Figure 3.20:](image)

**Figure 3.20:** Top left: Temporal evolution of the average vertical velocity through isosurfaces of constant temperature in the simulation box. Top right: Same as top left, but for particle flux. Bottom left: Net particle flux in temperature intervals with a spacing of 0.5 in $\log T$ as a function of simulation time. Bottom right: Same as bottom left, but for a spacing of 0.2 in $\log T$. 

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The weighting of the vertical velocity $v_z$ by the mass density $\rho$ of the plasma raised to the power of a positive integer $\alpha$ can be expressed by

$$\langle v \rangle_\alpha = \frac{\langle \rho^\alpha \cdot v_z \rangle}{\langle \rho^\alpha \rangle},$$

(3.6)

The $\alpha=0, 1, 2$ terms will be compared in the following. $\langle v \rangle_0 = \langle v_z \rangle$ is the spatially averaged vertical velocity, i.e. the average of all grid point velocities. $\langle v \rangle_1 = \frac{\langle \rho v_z \rangle}{\langle \rho \rangle}$ is the density-weighted velocity, i.e. the mean particle velocity. $\langle v \rangle_2 = \frac{\langle \rho^2 v_z \rangle}{\langle \rho^2 \rangle}$ is the intensity-weighted velocity. $\rho^2$ is proportional to the intensity of a spectral line (Eq. 2.50), and therefore $\langle v \rangle_2$ is expected to be related to the average Doppler velocities calculated from the line profiles (Eq. 2.75).

In Fig. 3.21, $\langle v \rangle_0$, $\langle v \rangle_1$ and $\langle v \rangle_2$ are investigated for the isosurfaces of temperatures between $\log T=4.0$ and $\log T=6.0$. The velocity-temperature relations for the timesteps $t=0, 10$ and $20$ min are shown, as well as the temporal averages. For comparison with the Doppler velocities, the velocity axis is plotted upside-down where negative velocities correspond to downflows (redshifts). The Doppler shifts calculated from the model are indicated by black diamonds in the lower right panel of Fig. 3.21 (data from Fig. 3.14). For temperatures above $\log T=6.0$ the small number of data points in the simulation box would result in unreliable predictions of the model which is why such temperatures are not considered in the following discussion.

The shape of the velocity versus temperature function derived from $\langle v \rangle_2$ is similar to that of the Doppler velocity function of the model (Fig. 3.21). The correlation between intensity-weighted velocities and Doppler shifts allows an interpretation of Doppler shifts in the form of the intensity-weighted velocities. Temporal variations of the shape of the velocity curves are small as can be seen when comparing the four panels of Fig. 3.21. All velocities are negative and show an increase in absolute value with temperature. The density-weighted ($\alpha=1$) velocities are larger than the spatially averaged ($\alpha=0$) velocities for temperatures higher than $\log T=5.0$, and the intensity-weighted ($\alpha=2$) velocities are larger than the density-weighted velocities. Therefore, an interpretation of Doppler shift as the average vertical velocity leads to a systematic overestimation of the mass flux.

It can be shown that even if there is only a small mass flux present expressed in the form of a small density-weighted velocity, the intensity-weighted velocities, i.e. the Doppler shifts, can be much larger. An illustration of this effect is depicted in Fig. 3.22. The density-weighted velocities and the intensity-weighted velocities of two loops with different densities $\rho$ and velocities $v$ in the loop legs are compared (Table 3.19). The parameters for the loops were chosen to represent both a mass flux balance (left; loop 1) and a net mass transport (right; loop 2). The intensity-
Figure 3.21: Vertical velocities through isosurfaces of temperature for timesteps $t=0$, 10, 20 min and temporal averages. The horizontally averaged ($\alpha=0$) velocities are indicated by dotted lines, the density-weighted ($\alpha=1$) velocities are indicated by dashed lines, and the intensity-weighted ($\alpha=2$) velocities are indicated by solid lines. Diamonds show the Doppler shifts of the model (compare Fig. 3.14).

The weighted velocity $\langle \rho^2 v \rangle / \langle \rho \rangle$ is calculated to be -0.4 for loop 1 indicating a downflow of material in the right leg, even though the density-weighted velocity according to $\langle \rho v \rangle / \langle \rho \rangle$ is zero and no mass flux is present in this loop. In a similar way, loop 2 shows an intensity-weighted velocity of -1.8, larger than the density-weighted velocity of -1.2. Both examples show that the interpretation of the intensity-weighted velocity in terms of a mass flux would result in an overestimation of the mass downflux from the observed Doppler redshifts.
3.4 Mass Balance in the Corona

Figure 3.22: Left: Coronal loop (loop 1) showing a mass flux balance. Right: Coronal loop (loop 2) showing a net mass transport. The length and direction of the arrows indicates the strength and direction of the velocity in the loop legs. The thickness of the arrows indicates the density. The values are listed in Table 3.2.

<table>
<thead>
<tr>
<th></th>
<th>loop 1</th>
<th></th>
<th>loop 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>left</td>
<td>right</td>
<td>left</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$v$</td>
<td>2</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>$\langle \rho \cdot v \rangle$</td>
<td>0</td>
<td>-1.2</td>
<td></td>
</tr>
<tr>
<td>$\langle \rho^2 \cdot v \rangle$</td>
<td>-0.4</td>
<td>-1.8</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Values for the densities and velocities of the coronal loops (loop 1 and loop 2) shown in Fig. 3.22. Rows three and four show the respective density-weighted velocities $\langle \rho \cdot v \rangle / \langle \rho \rangle$ and the respective intensity-weighted velocities $\langle \rho^2 \cdot v \rangle / \langle \rho^2 \rangle$. 
3.5 Investigation of Coronal Loop Flows

In this section fieldlines are traced in the simulation box with time, and plasma flows are analyzed along these lines. The method is outlined in section 3.5.1 and in section 3.5.2 coronal loop flows are presented. A new scenario for a chromosphere-corona mass cycle is introduced in section 3.5.3.

In general, a magnetic field line is a mathematical construct with no physical meaning. However, in the case of ideal magnetohydrodynamics that can be applied for the solar atmosphere, a field line can be regarded as a material line moving with the plasma (section 2.2.1.2).

3.5.1 Tracing Magnetic Field Lines in the Simulation Box

For the temporal tracing of magnetic field lines in the simulation box two different techniques are applied. The first technique is the field line advection routine included in the VAPOR (Visualization and Analysis Platform for Ocean, Atmosphere, and Solar Researchers) software package (Clyne and Rast, 2005), (Clyne et al., 2007). This routine animates the steady magnetic field lines that are subject to a time-varying velocity field. Each field line is determined from the corresponding field line at the previous time step, and as a result a set of different steady flow lines is obtained at each time step. Alternatively, a routine has been developed in which the field lines are being advected by the plasma. In the solar atmosphere, magnetic field lines are “frozen in” the plasma, i.e. they behave as if they moved with the plasma (section 2.2.1.2). Therefore, the velocity vector of the plasma at the corresponding timestep and site (i.e. the top of the field lines) can be used to determine the displacement of the starting points of the fieldlines for the next timestep. The highest point of each field line is displaced because the field lines are most widely separated from each other higher up in the atmosphere. Thus, errors resulting from jumping between different field lines can be minimized. The magnetic field vector is tracked in three dimensions from the starting point until the field line returns to the bottom or leaves the simulation box at the top. Field lines that leave the simulation box at the side can still be tracked as periodic boundary conditions are applied in the horizontal direction.

The field line tracking is started in the regions of strong Doppler shifts ($v_D < -8 \text{ km/s}$) and in the layer of highest emission of the C\textsc{iv} (1548 Å) line in the simulation box. A set of approximately 200 field lines is investigated. Both the VAPOR field line advection routine and the user-written fieldline routine lead to similar results for the temporal field line evolution.
3.5.2 Analysis of Loop Flows

In Fig. 3.23 the time-dependent evolution of two field lines, one lying on top of the other, is shown for a solar time period of 30 minutes. The blue and red colors of the field lines indicate the vertical velocity along the field lines. The velocities are scaled from -1 km/s (deep red) to +1 km/s (deep blue), where red colors indicate downflows \((v_z < 0)\) and blue colors indicate upflows \((v_z > 0)\) at the respective sites. The field lines are shown from three different perspectives. The left and middle panels show the configuration of the field lines when looking at the simulation box from the side; the right panels show the field lines when looking at the box from above.

In the following, reference is made to the yz-plots of Fig. 3.23 (middle column). The right footpoint of the large field line changes its position between \(t=1\) min and \(t=2\) min and return to this location between \(t=19\) min and \(t=20\) min. The left footpoint of the small field line migrates to left footpoint of the large field line between \(t=6\) min and \(t=7\) min. The right footpoint of the small line changes its location from the far end of the y-axis to zero between \(t=11\) min and \(t=12\) min. Between \(t=17\) min and \(t=18\) min this footpoint migrates back to the middle of the box. This sort of reconnection phenomenon is found more or less for almost all of the field lines of the examined ensemble. The next section will show that the changing magnetic structure is associated with a change of the velocity direction of the plasma.

3.5.3 Chromosphere-Corona Mass Cycle

During a reconnection process antiparallel field lines initially disconnected from another can “reconnect” and form a new, continuous connection pattern. A highly idealized model of this reconnection process is sketched in Fig. 3.24 (left). The main features of this model are the advection of antiparallel field lines and plasma toward a dissipative region around the so-called “X-type neutral point” and their reconnection to form field lines that are carried out of this region.

Such a “X-type reconnection” site happens to exist in the simulation box. A nearby field line is selected for further analysis and its time-dependent evolution is shown in the left panel of Fig. 3.25 for three timesteps between \(t=8\) min and \(t=10\) min simulation time. During the course of the simulation the “left” footpoint of the field line remains connected to the same photospheric region, whereas the “right” footpoint moves to a different photospheric region near the center of the box between \(t=8\) min and \(t=9\) min simulation time.
3 Results of a High-Resolution Time-Series
3.5 Investigation of Coronal Loop Flows

![Figure 3.23](image.png)

Figure 3.23: Temporal evolution of two fieldlines one lying on top of the other at t=0 min. The blue and red colors indicate the vertical velocity ($v_z$) along the field lines. The velocities are scaled from -1 km/s (red, representing downflows) to +1 km/s (blue, representing upflows). The timesteps shown are those for which the field lines’ footpoints change their connectivity.
The vertical velocity along the loop is analyzed at each timestep and the vertical velocity at the location of highest C\textsc{iv} (1548 Å) intensity along the field line is shown in the right panel of Fig. 3.25. A quantitative analysis of the velocities shows a change in sign as the field line reconnects (from black to dark gray, Fig. 3.25). In a first phase (t=0-8 min) a positive velocity, corresponding to an upflow of material, is observed at the location of highest intensity along the line. As the field line reconnects, the vertical velocity switches its sign. During the phase when the field line is connected to another region plasma is draining out of the loop. The field line changes its connectivity again and reconnects for the second time between t=12 min and t=13 min (not shown). The vertical velocity also switches sign and is found to be positive again corresponding to an upflow of material within the loop. The vertical velocities of two spatially fixed regions where mass loading ($v_z > 0$ at t=6 min) and draining ($v_z < 0$ at t=10 min) take place are indicated by the dotted lines in the right panel of Fig. 3.25. The temporal evolution of these velocities shows little variation and indicates that these sites in the simulation box remain rather stable for the whole simulation run.

Figure 3.24: Left: The magnetic field lines near an X-type neutral point. A configuration for steady magnetic reconnection adapted from Priest (1984). Right: Configuration of field-lines near X-type neutral point. (1) and (2) denote regions of upflow (blue) and downflow (red).

A similar behavior is observed for many other magnetic field lines of the ensemble. Two examples for the temporal evolution of the vertical velocities of field lines are presented in Fig. 3.26. In addition to the sites where the C\textsc{iv} (1548 Å) intensity is highest along the field line, the vertical velocities at the positions of highest C\textsc{ii} (1335 Å) and C\textsc{iii} (977 Å) intensity along the field line are shown. The timesteps where the field lines change their connectivity are indicated by vertical dashed lines.
Whenever a field line switches its footpoint, the vertical velocity at the reconnection site changes sign. The blue and red dots indicate the strength of the observed velocities that have been scaled from -1 km/s (downflows) to +1 km/s (upflows). Another indication for the filling of loops during a heating phase and for the draining of loops during a cooling phase is illustrated in Fig. 3.27. The left panel shows the decrease in temperature along the loop in the draining phase between $t=2$ min and $t=10$ min. The right panel of Fig. 3.27 shows the increase in temperature during the phase where material is filled into the loop between $t=20$ min and $t=30$ min.

According to the description above, spatially confined upflow regions that are intermittently connected to coronal loops lead to mass loading of the loop. After reconnection the loop starts to drain and material is flowing out of the loop. Fig. 3.24 shows a two-dimensional sketch of the reconnection site. The situation before reconnection takes place is indicated by the blue arrow labeled (1) indicating upflows (visible as blueshifts) along the field line. The situation after the reconnection has taken place is indicated by the red arrow labeled (2) indicating downflows (visible as redshifts) along the field line. The right panel shows the situation when looking at the reconnection site from above. The reconnection point is marked by a cross in the middle of the plot. Oppositely directed field lines that are “frozen into” the plasma are carried towards another by the plasma flow indicated by the black arrows. (1) and (2) correspond to the upflow and downflow regions in the left panel and show always the same properties, e.g. upflow or downflow velocities. The field lines are intermittently connected to different photospheric regions and therefore alternately
3 Results of a High-Resolution Time-Series

Figure 3.26: Examples of vertical velocities along reconnecting magnetic field lines at sites of highest \( \text{C} \, \text{II} \) (black), \( \text{C} \, \text{III} \) (gray) and \( \text{C} \, \text{IV} \) (light gray) emission. Blue and red colors indicate the vertical velocity ranging from -1 km/s (red, downflows) to +1 km/s (blue, upflows). The left example shows the large loop discussed in section 3.5.2.

Figure 3.27: Temporal evolution of the temperatures along the loop (same loop as in Fig. 3.26, left) during the draining phase (t=2-10 min; left) and during the mass inflow phase (t=20-30 min; right).

traverse regions of upflows and downflows.
4 Results for a Transient Ejection

In this chapter a second numerical simulation run of the atmosphere above a small active region is analyzed. The setup is similar to the previous time-series (chapter 3), e.g. similar boundary conditions (see section 2.1) have been applied. Details of the MHD model can be found in Bingert (2009). Rather than 30 minutes in the first run this time-series covers 71 minutes of solar time with a temporal resolution of 30 seconds. The spatial resolution of this run is $128^3$ grid points.

Of particular interest is an eruption in the form of a plasma blob, that occurs in the middle of the time-series. The formation of the plasma blob and the time-dependent evolution are analyzed in detail in section 4.1. Results for the mass balance in the simulation box are presented in section 4.1.5. In section 4.2 spectra of ultraviolet emission lines synthesized from these data are investigated.

4.1 The Plasma Eruption

A transient ejection of plasma in the form of a plasma blob eruption is being observed between approximately $t=39$ min and $t=56$ min of simulation time. Fig. 4.1 shows images of the synthesized emission of O vi (1032 Å) integrated along the horizontal and vertical directions and the corresponding Doppler shift maps that have been scaled from -10 km/s (blue) to +10 km/s (red). The intensity and Doppler shift maps will be described in detail in sections 4.2.3 and 4.2.4. When viewing the O vi (1032 Å) line with regard to the intensity, the blob appears bright in either direction (Fig. 4.1, columns one to three). The rise and subsequent descent of material are evident as redshifts and blueshifts in the Doppler maps (Fig. 4.1, column four).

In 4.1.1 the onset of the plasma eruption will be analyzed. In 4.1.2 the blob will be characterized in detail with regard to parameters like volume, mass, temperature and density. In section 4.1.3 typical parameters will be determined in blob locations that are typical for its front region and its center-of-gravity. In section 4.1.4 the blob is investigated with regard to adiabatic expansion and finally, in section 4.1.5 the mass increase of the blob is examined.
4 Results for a Transient Ejection
4.1 The Plasma Eruption

Figure 4.1: Time-series of O VI (1032 Å) emissivities integrated along x- (left), y- (middle left) and z-direction (middle right) of the box together with Doppler shift maps (right) for different timesteps during the plasma eruption (t=39-56 min). The integrated emissivities are logarithmically scaled from 0.1 to 10 with respect to the mean value of the integrated emissivity at timestep t=0 min. The Doppler shift maps are scaled from -10 km/s (blue; upflows) to +10 km/s (red; downflows).
4 Results for a Transient Ejection

4.1.1 The Onset of the Plasma Eruption

Since the area of the blob's origin is hidden in the bright shining plasma (Fig. 4.1), the determination of the onset of the plasma eruption in terms of time and location is difficult to perform based on the synthesized emission. However, when investigating the plasma parameters in detail, anomalies in the profiles are found that provide an indication of where and when the blob starts to form. Fig. 4.2 shows the time-dependent evolution of temperature, density, pressure and current density squared in small volumes indicated by colored boxes in the intensity images of O VI (1032 Å) in the bottom row of the figure. The red box was chosen to capture the formation region of the blob. The green and yellow boxes are the areas where the blob appears above the bright shining plasma between $t=39$ and $t=40$ min. Prior to the eruption of the blob, a drop in temperature and an increase in density is observed in the regions where the blob starts to form (Fig. 4.2, top row). This results in a pressure increase (Fig. 4.2, middle left). At the same time the heating ($\propto j^2$) continues to increase (Fig. 4.2, middle right) in these regions.

Fig. 4.3 (a) shows the time-dependent evolution of the vertical pressure gradient in the blob formation regions indicated by the colored boxes of Fig. 4.2. The mean pressure in the vertical red column of Fig. 4.2 is shown as a function of height in the simulation box in Fig. 4.3 (b). The dashed lines indicate the extension of the red box in the vertical direction. Panel c shows the mean pressure in the yellow/green column of Fig. 4.2. Panels d, e and f show the average heating rate ($\propto j^2$), the average temperature and the average density in the vertical red column as a function of height in the box. The pressure gradient abruptly peaks in the red box at $t=37$ min. Sudden descents are observed in the red, yellow and green boxes between $t=38.5$ min and $t=39$ min (Fig. 4.3, a). Prior to the pressure gradient peak, a slow decrease of the pressure gradient is observed in the area where the blob starts to form, i.e. in the red box. At about $t=40$ min, all pressure gradients are back to the initial height. The decrease of the pressure gradient at $t=33$ min is the earliest indication of the blob formation (panel a). Starting at the same time, a pressure disturbance is observed at a height of approximately 5 Mm that intensifies subsequently between $t=32$ min and $t=37$ min (Fig. 4.3, b). At $t=37.5$ min, a solitary perturbation is moving upwards in the simulation box. The part of the pressure disturbance moving downwards is effectively dissipated by the dense plasma of the low atmospheric layers and can therefore not be observed. Typically, a disturbance that

\footnote{A solitary wave, or soliton, is a special kind of surface wave that preserves its original shape as it propagates. An illustrative example of solitary waves are surface waves in canals. In circumstances where $k\zeta$ and $\zeta/d$ are both significant but less than unity ($|\zeta| \approx \lambda \approx d$, $\zeta$ elevation of the surface, $d$ depth of water, $\lambda$ wavelength), the two tendencies of a wave, dispersion and shock formation compensate each other (Fabrikant, 1995). A number of non-linear differential equations, such as the Korteweg-de Vries (KdV) equation have solitary wave solutions. Solitary perturbations of magnetic flux tubes on the Sun have been studied by e.g. Stix et al. (2005).}
travels upwards into regions of lower gravity and density steepens and eventually forms a shock. However, dissipation can counteract this tendency to form a shock and thus the shape of the perturbation will be invariant on its way upwards in the simulation box. This phenomenon of a soliton-like structure is being observed here. The soliton-like structure keeps its amplitude and half width and leaves the top of the red box at $t=38.5$ min. At $t=39$ min, the solitary perturbation has traveled up to approximately 6.5 Mm (panel b). It is now also seen in the yellow/green column.

Figure 4.2: Temporal evolution of temperature, density, pressure and current density squared in small volumes shown as green, yellow and red boxes in the OVI (1032 Å) intensity images in the bottom row. The colors correspond to the box colors.
4 Results for a Transient Ejection

(panel c) as a pressure disturbance of constant amplitude and half width. As the leading edge of the soliton-like structure enters the yellow and green boxes (panel c), an abrupt decrease of the pressure gradient is observed (panel a). In contrast, as the trailing edge of the upward moving soliton-like structure is about to leave at top of the box, a positive pressure gradient is observed.

The time-dependent evolution of the heating rate ($\propto j^2$) in the red column shows a continuous increase between $t=10$ min and $t=40$ min (panel d) at all heights. This issue will be further investigated in section 4.2.5. As a consequence, the time-dependent evolution of the temperature shows a rise between $t=10$ min and $t=35$ min (panel e). The time-dependent evolution of the density shows a decrease in the blob formation region before the ejection (panel f).

Fig. 4.4 shows the horizontal variation of the pressure, temperature, density and heating rate along the horizontal column that contains the blob formation region (compare Fig. 4.2, red dashed lines). Other than in Fig. 4.3, the variations of these parameters are considerable across the horizontal planes in Fig. 4.4. The formation region is found to be close to a pressure minimum and surrounded by strong horizontal pressure gradients (Fig. 4.4 a and b) before the formation of the blob sets in. These appear to be necessary conditions for the increase of the pressure that occurs prior to the rise of the blob. But most likely these conditions are not responsible for triggering the plasma ejection. The reason for this assumption is that the plasma in the blob formation region is in a low-$\beta$ plasma state (section 4.2.2) and therefore, motions are constrained along the (vertical) magnetic field. Thus, horizontal pressure gradients will not have a great effect on the acceleration of the blob.

A noticeable feature in the blob formation region during the phase preceding the plasma ejection is the time-dependent evolution of the temperature. Shortly before the ejection takes place, the temperature has climbed up to almost one million Kelvin compared to the surrounding temperature of $T = 10^4 - 5 \times 10^4$ K (Fig. 4.4 c and d). The density profiles show a local minimum in both the x- and y-direction at the blob formation site (panels d and e) leading to the aforementioned pressure decrease. A significant local increase of the heating rate is observed at the formation site (panels g and h). As a consequence, the temperature increases at that site (see panels b and c).

The plasma blob finally emerges above the bright plasma at a height of approximately 10.5 Mm between $t=39$ min and $t=40$ min (Fig. 4.1) reaching its maximum height of 21.5 Mm at $t=46$ min. The blob shows a dynamic shape; it appears tailed when ascending and descending and ellipsoidal when being close to its maximum height. The blob returns to the initial height at $t=51.5$ min. Subsequently, it slowly disappears in the bright plasma and is completely disappeared after $t=56$ min. An
4.1 The Plasma Eruption

Figure 4.3: a: Time-dependent evolution of the mean pressure gradient in small volumes. The colors correspond to the box colors in Fig. 4.2. b,c: Mean pressure as a function of height in the simulation box for red and yellow/green columns. d: Current density squared along vertical red column. e,f: Same as panel d, but for temperature and density. The legend that corresponds to panels d, e and f has been shifted to the bottom of the plot.

upwards moving cooling front is observed in the tail (Fig. 4.3 bottom row).

4.1.2 Time-Dependent Evolution of Average Parameters of the Plasma Eruption

In order to characterize the blob in more detail, it is necessary to identify the gridpoints that are passed by the blob. Criteria for the identification of the gridpoints are determined to be an emissivity greater than $10^{-7}$ W/m$^3$ in the respective
4 Results for a Transient Ejection

Figure 4.4: Time-dependent evolution of the mean pressure (panels a, b), temperature (panels c, d), density (panels e, f) and current density squared (panels g, h) in a horizontal column through the red box (Fig. 4.2) as a function of the horizontal directions x (left) and y (right). Timesteps are indicated in the legend at the top of the figure. The position and width of the box is shown by red dashed lines. See section 4.1.1.

line (CII (1335 Å), CIV (1548 Å), Ovi (1032 Å)), a particle density higher than \( n = 2 \times 10^{14} \text{ m}^{-3} \), as well as a temperature higher than \( T = 6 \times 10^5 \text{ K} \). It must be noted that these criteria will not be able to catch all gridpoints that are temporarily
4.1 The Plasma Eruption

Figure 4.5: Highly-interpolated O\textsc{vi} (1032 Å) emissivities (courtesy of Hardi Peter) integrated along the y-direction of the simulation box for timesteps t=48, 49, 50, 51, 53, 54, 57.5, 58.5 min (xz-view). The size of the simulation box is 50 Mm in the horizontal direction and 30 Mm in the vertical direction (compare Fig. 4.1).

part of the blob. There are various reasons for this, e.g. the temperature threshold will not be reached in the center of the blob. However, the above criteria are sufficient to visualize the blob contours at least. These contours are afterwards filled with gridpoints that initially were not identified by the chosen criteria.
Volume, mass and the means of density, velocity and pressure are determined at all timesteps between $t=40$ min and $t=51.5$ min, as are the means of plasma-$\beta$ and plasma-$\beta_{\text{kin}}$ of the blob (Fig. 4.6). The parameters are based on the appearance of the blob in the C$\text{\textsc{ii}}$ (1335 Å), C$\text{\textsc{iv}}$ (1548 Å) and O$\text{\textsc{vi}}$ (1032 Å) lines. Due to the decrease in pressure with height the blob expands when ascending and contracts on its way back (Fig. 4.6, panel a). When being viewed in the O$\text{\textsc{vi}}$ (1032 Å) line, the extension of the blob is larger than when being viewed in the lines C$\text{\textsc{ii}}$ (1335 Å) and C$\text{\textsc{iv}}$ (1548 Å). Since the blob is hotter outside, these regions can only be observed in the O$\text{\textsc{vi}}$ line (line formation temperature $T_f = 3 \times 10^5$ K). In the C$\text{\textsc{ii}}$ ($T_f = 4 \times 10^4$ K) and C$\text{\textsc{iv}}$ ($T_f = 10^5$ K) lines these regions do not contribute to the blob, which is why the blob appears to be smaller when observing these lines. It should be noted that the blob extension and the mean temperature are sensitive to the emissivity threshold. Both curves are merely shifted up and down, whereas the shape of the curves remains the same.

The average temperature (Fig. 4.6, middle left) of the plasma blob drops by a approximately a factor of two during the rising phase and shows an increase during the sinking phase. The mass and the density of the blob continuously increase (upper right, middle right). The blob’s highest mean density is found when the blob is observed in the C$\text{\textsc{ii}}$ (1335 Å) line, whereas the blob’s highest mean temperature is found for viewing the blob in the O$\text{\textsc{vi}}$ (1032 Å) line. The pressure being proportional to the product of both density and temperature, is similar for C$\text{\textsc{ii}}$, C$\text{\textsc{iv}}$ and O$\text{\textsc{vi}}$. Plasma-$\beta_{\text{kin}}$ is larger than plasma-$\beta$ for all three lines (bottom right) which indicates that the kinetic energy density of the plasma dominates the thermal energy density inside the blob, especially shortly after its ejection when the velocity of the blob is large.

### 4.1.3 Evolution of the Center-of-Gravity and the Front of the Plasma Eruption

In this section parameters like the temperature and velocity etc. will be determined in blob areas typical for its front region and its center-of-gravity for every timestep. As a first step the center-of-gravity is determined by averaging the locations of all blob grid points taking into consideration the individual particle densities at these points. The center-of-gravity is tracked in the time-period $t=40-51.5$ min when the entire blob is visible. The front of the blob is defined to be the intersection point of the line connecting consecutive center-of-gravity points and the outermost layer of the blob. The front of the blob is tracked in the time-period $t=39-51$ min when the front is visible.
4.1 The Plasma Eruption

4.1.3.1 Temperatures and Densities

The temperatures and densities at the front of the blob and at the center-of-gravity are considerably different (Fig. 4.7, top row). Temperatures at the front of the blob reach values of up to one million Kelvin, whereas temperatures inside the blob are as low as a few thousand Kelvin (upper left). Differences of up to four orders in magnitude between the front and the center-of-gravity are also observed for the particle densities of the blob (upper right) indicating a strong variation of these...
4 Results for a Transient Ejection

parameters inside the blob. Cuts through the blob (Fig. 4.8) along the x-axis at the center-of-gravity of the blob show a drop in temperature from $T = 2 \times 10^5$ K down to $T = 900$ K. The cuts also show a particle density increase from $n = 10^{15} \text{ m}^{-3}$ up to $n = 2 \times 10^{17} \text{ m}^{-3}$ inside the blob.

4.1.3.2 Plasma-$\beta$

The ratio of kinetic energy density and magnetic energy density $\beta_{\text{kin}}$, as well as the ratio of thermal energy density and magnetic energy density $\beta$ for the front, the center-of-gravity and the entire blob are less than one (see Fig. 4.7, middle right). The blob front is the hottest region and therefore has a high thermal energy density which is why plasma-$\beta$ is larger than plasma-$\beta_{\text{kin}}$. In the center of the blob, the temperature is lower than on the outside, and consequently the plasma-$\beta$ plasma-$\beta_{\text{kin}}$ relation is reversed. As can be seen, $\beta < \beta_{\text{kin}}$ when averaged over the entire blob. This is confirmed by Fig. 4.8 which shows that $\beta < \beta_{\text{kin}}$ for most parts inside the blob.

4.1.3.3 Vertical Velocities

The vertical plasma velocities $v_z$ of the center-of-gravity and of the front are shown in the lower left panel of Fig. 4.7. In addition, the vertical offset velocities $dv_z$, i.e. the vertical distance of consecutive blob positions divided by the time interval 30 seconds, are indicated by dashed lines. Both velocities are similar at the front and at the center-of-gravity which indicates that the blob is a moving plasma drop rather than some kind of wave phenomenon traveling through the box. When moving upwards, the center-of-gravity is faster than the front which corresponds to the change of the blob’s shape from tailed to ellipsoidal (Fig. 4.1). The blob extends again as it falls down, and at about $t=46.5 \text{ min}$ the front is decelerated. This is due to the deformation of the magnetic field by the blob as will be explained in section 4.1.3.4.

The ejection in the form of a plasma blob appears to be a hydrodynamic effect rather than a phenomenon associated with the changing magnetic field, e.g. a jet. The determination of the velocity at the start of the ejection shows that the vertical velocity of the plasma blob, i.e. $v_z \approx 50 \text{ km/s}$, is of the order of the sound speed ($c_s = 152 T^{1/2} \text{ m/s} \approx 50 \text{ km/s}$ at $T = 10^5 \text{ K}$ according to [Priest, 1984]) and corresponds to only $1/6$ of the Alfvén speed ($v_A = 2.8 \times 10^{12} B \cdot n^{-1/2} \text{ m/s} \approx 300 \text{ km/s}$ [Priest, 1984], assuming typical coronal values of magnetic field strength $B = 1 \text{ G}$ and particle density $n = 10^{14} \text{ m}^{-3}$).

Table 4.1 shows the accelerations of the blob’s front and center-of-gravity that are calculated from the vertical velocities of Fig. 4.7. Linear fits to the velocities are performed in the time-periods indicated in Table 4.1. During the rising phase
4.1 The Plasma Eruption

(t=39.5-44 min) the front is decelerated by $a=255 \text{ m/s}^2$, almost as high as expected in view of the gravitational acceleration $g_{\text{Sun}}=274 \text{ m/s}^2$. The center-of-gravity is decelerated less than the front ($a=228 \text{ m/s}^2$ for $t=41-45$ min), while the shape of the blob turns from tailed to ellipsoidal (Fig. 4.1). During the blob’s fall the acceleration of the center-of-gravity is below gravitational acceleration indicating that the blob looses kinetic energy while performing work against the magnetic field. A linear fit to the velocities of the front in the falling phase is not meaningful, since the front is not decelerated constantly (see Fig. 4.7).

<table>
<thead>
<tr>
<th>region of the blob</th>
<th>phase [min]</th>
<th>$a$ [m/s$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>front</td>
<td>39.5 - 44.0</td>
<td>$255 \pm 12$</td>
</tr>
<tr>
<td>center-of-gravity</td>
<td>41.0 - 45.0</td>
<td>$228 \pm 8$</td>
</tr>
<tr>
<td>center-of-gravity</td>
<td>45.5 - 49.5</td>
<td>$174 \pm 4$</td>
</tr>
</tbody>
</table>

Table 4.1: Vertical acceleration of the blob’s front and the center-of-gravity. The values are derived by applying linear fits to the vertical velocities functions of Fig. 4.7. $1\sigma$ uncertainty estimates of the vertical acceleration are also shown.

The blob returns from a height of 21.5 Mm speeding up to $v_z = 55 \text{ km/s}$ at the center-of-gravity and to $v_z = 40 \text{ km/s}$ at the front. It disappears into the bright shining plasma at a height of approximately 10.5 Mm (Fig. 4.7 lower right), where the theoretical free-fall speed is 78 km/s (Fig. 4.8 lower right). As can be seen from the figure, the vertical velocities of both the center-of-gravity of the blob and of the front are close to the free-fall speed at the beginning of the fall. But soon the blob is decelerated and the different parts of the blob move at a speed smaller than the free-fall speed. The front of the blob is moving at a speed below the local sound speed at all timesteps, and thus the formation of a shock front is unlikely.

4.1.3.4 Horizontal Velocities

The center-of-gravity of the blob moves at almost constant horizontal velocity up to $t=48$ min (Fig. 4.7 lower right). By this time the blob has moved beyond its maximum height and is already returning. At $t=48$ min the angle between velocity vector and magnetic field vector shows a maximum at both the center-of-gravity and the front (Fig. 4.9 left), and the blob starts slowing down (Fig. 4.7 lower right).

$^2$The sound speed $c_s = \sqrt{\frac{\gamma T}{m_{\text{p}}}} = \sqrt{\frac{E}{m}}$ can be approximated by $c_s = 152 T_0^{1/2}$ m/s, where $\gamma$ is the ratio of specific heat at constant pressure $c_p$ and constant volume $c_v$ (Priest and Forbes, 2000). In the transition region ($T_0 = 10^5 \text{ K}$) and corona ($T_0 = 10^6 \text{ K}$) the sound speed takes typical values of $c_s \approx 50 \text{ km/s}$ and $c_s \approx 150 \text{ km/s}$. 

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4 Results for a Transient Ejection

Fig. 4.10 shows the configuration of magnetic field lines connecting regions of opposite polarity before the ejection of plasma takes place (t=30 min). The migrating center-of-gravity of the blob is indicated by a red line. As expected, the blob is moving parallel to the magnetic field lines during its rising phase - as can be seen from figures 4.9 (left) and 4.11. When the blob turns back, it deforms the magnetic field dynamically. This deformation of the magnetic field is confirmed by the observation of a higher current density at the front region of the blob which can only be a consequence of the magnetic field lines being squeezed together (see section 4.2.5). This effect is also observed as a brightening of the front when looking at the line intensities (see Fig. 4.3). In the right panel of Fig. 4.9 the plasma-β values of the front and the center-of-gravity are shown. Maxima are found at roughly the same time interval for both the front angle and front plasma-β functions.

4.1.4 Adiabatic Expansion

Since the blob volume increases rapidly after the onset of the eruption (Fig. 4.6 upper right), it will be examined whether or not the blob volume complies with adiabatic expansion. The volume of the blob in the O vi (1032 Å) line increases by a factor of 4.6 within three minutes (t=40-43 min) and in parallel, the temperature of the center-of-gravity drops by a factor of 3.3 (Fig. 4.7). The change of the inner energy $U = n \cdot k_B \cdot T / (\kappa - 1)$ for an adiabatic process is $dU = dq - p dV = -pdV$, where $\kappa = 1.67$ is the adiabatic coefficient of a monoatomic gas. $T$, $n$ and $V$ are the respective temperature, particle density and volume. The inner energy of the blob increases by approximately 40% between $t=40$ min and $t=43$ min. For an adiabatic process, the product $T \cdot V^{\kappa - 1}$ should be constant. When typical values for the blob are considered, $T \cdot V^{\kappa - 1}$ is 20% smaller at $t=40$ min than at $t=43$ min. This means that the temperature of the blob decreases more than expected for adiabatic expansion in that phase. A possible explanation for this observation may be that heat is being transferred to the surroundings, e.g. due to radiative losses or viscous heating at the front. One should also note that the average blob parameters depend on the criteria applied for the blob detection (section 4.1.2).

4.1.5 Mass Balance

The mass of the blob increases linearly during its flight (see Fig. 4.6 upper right). 11 minutes after its start the blob is 2.6 times heavier than at the start. The mass picked up by the blob is approximately $10^9$ kg (Table 4.2) and corresponds to 25% of the total coronal mass that has been defined as the mass in the box above $log T = 6.0$. However, the mass inside the volume of the tube passed by the blob would not be able to contribute more than 4% of the mass increase of the blob (Table 4.2). This mass discrepancy will be explained in the following.
4.1 The Plasma Eruption

Prior to the rise of the blob and during the flight of the blob the total mass of the entire box is almost constant (Fig. 4.11, upper left). On the other hand, variations of 25% are observed between t=40 min and t=51 min, when only the coronal mass of the box (above 11.8 Mm corresponding to an average temperature of $\log T = 6.0$) is considered (upper right). The coronal mass accounts for 0.0000015% of the total mass of the box in the phase prior to the rise of the blob and for 0.0000019% of the total mass of the box while the blob exists (middle left). Therefore, mass variations
Figure 4.8: *Top row:* Variation of temperature and density along the x-axis at the center-of-gravity of the blob for timestep $t=43$ min. *Lower left:* Same as top, but for plasma-$\beta$ and plasma-$\beta_{kin}$. The dotted lines indicate the center-of-gravity of the blob along the x-axis. *Lower right:* Comparison of free fall speed (solid line) with the center-of-gravity velocity (dotted line) and velocity of the front of the blob (dashed line). See section 4.1.3.

due to the presence of the blob are neglectable compared to the mass of the entire box. In the panel in the middle right of Fig. 4.11, the mass of the corona including the volume of the tube that the blob is passing and the mass of the corona without this tube are compared. Both masses are very similar for most of the time except for the phase when the blob appears in the corona ($t=40$-51 min).

The mass of the tube (difference of the corona with versus without tube) is nearly constant prior to the rise of the blob and after the disappearance of the blob (Fig. 4.11 bottom left). As a consequence, changes in coronal mass are caused by the mass of the blob that is passing through the box. The comparison between the mass of the tube and the mass of the blob shows that the mass of the tube grows even faster than that of the blob as the blob passes through the tube (bottom left). In addition, a cut through the tube at a height of 10.5 Mm (height where the blob
4.1 The Plasma Eruption

Figure 4.9: Left: Time-dependent evolution of the angle between the velocity vector and the vector of magnetic field strength at center-of-gravity (cog; light gray) and front (black) of the blob. Right: Time-dependent evolution of plasma-β at the front and the center-of-gravity of the blob (compare middle right panel of Fig. 4.7). The dashed lines in both graphs indicate the timesteps (t=44 min and t=46 min), where the front and the center-of-gravity of the blob reach their maximum heights in the simulation box.

Figure 4.10: Configuration of magnetic field lines connecting regions of opposite magnetic flux at t=30 min. The field lines are traced in regions where the blob appears and disappears. The red line shows the migrating center-of-gravity of the blob. Left: Viewing the simulation box from above. Right: Viewing the simulation box from the side at an angle of 30 degrees.
4 Results for a Transient Ejection

Table 4.2: Mass of plasma blob as it appears in the C\textsc{ii} (1335 Å), C\textsc{iv} (1548 Å) and O\textsc{vi} (1032 Å) lines at start (t=40 min) and end (t=51 min) of its flight time. The column mass\textsubscript{tube} shows the average mass of the tube that the blob passes between t=15 min and t=30 min, before the plasma eruption takes place. The right column shows the fraction of the total mass of both the blob at t=40 min and the entire “empty” tube divided by the mass of the blob at t=51 min.

<table>
<thead>
<tr>
<th>line</th>
<th>mass\textsubscript{blob} [kg]</th>
<th>mass\textsubscript{tube} [kg]</th>
<th>\frac{mass\textsubscript{tube}+mass\textsubscript{blob}(t=40min)}{mass\textsubscript{blob}(t=51min)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C\textsc{ii}</td>
<td>6.08 \times 10^8</td>
<td>1.60 \times 10^9</td>
<td>4.90 \times 10^8</td>
</tr>
<tr>
<td>C\textsc{iv}</td>
<td>6.12 \times 10^8</td>
<td>1.61 \times 10^9</td>
<td>5.85 \times 10^7</td>
</tr>
<tr>
<td>O\textsc{vi}</td>
<td>6.14 \times 10^8</td>
<td>1.62 \times 10^9</td>
<td>6.85 \times 10^7</td>
</tr>
</tbody>
</table>

appears) shows that a mass of $1.5 \times 10^9$ kg flows into the tube between t=39.5 min and t=40 min (bottom right). This mass is already sufficient to explain the final mass of the blob at the end of its flight and indicates that the blob does not simply deplete the entire tube as it passes through collecting all the material along its way. Rather, the material is already present in the tube at t=40 min, and even more material continues to flow into the tube after t=40 min. A possible explanation for the observed mass increase of the blob (Fig. 4.6, upper right) could be the deficiency of the identification method to detect the entire blob right from the start.
Figure 4.11: Top left: Mass balance for entire box as a function of simulation time. Top right: Mass balance for coronal part of the box (above 11.8 Mm corresponding to an average temperature $\log T = 6.0$) as a function of simulation time. Middle left: Temporal evolution of the ratio of coronal mass and total mass of the box. Middle right: Comparison between coronal mass with blob (black) and coronal mass without blob (gray). Bottom left: Same as middle right, and comparison between the mass of the tube that the blob passes (dashed line) and the mass of the blob (dotted line). Bottom right: Mass flowing through a horizontal cut of the tube at 10.5 Mm height. Only the left side of the tube has been considered for this estimate.
4 Results for a Transient Ejection

4.2 The Model Atmosphere

4.2.1 Temperature, Density, Velocity and Magnetic Field Strength

The investigation of the atmospheric parameters is performed in a similar way to that of section 3.1, where a detailed description of the procedure is given. Fig. 4.12 shows the average temperature $T$, particle density $n$, vertical velocity $v_z$ and magnetic field strength $B$ as a function of height in the simulation box. First, horizontal averages of $\log T$, $\log n$, $v_z$ and $|B|$ are calculated at each timestep. In a second step, temporal averages of these values are determined at each height for the entire time-series. Both the steep rise in temperature in the transition region and the exponential decay of the magnetic field strength in the corona are seen in Fig. 4.12 (compare Fig. 1.3 and Fig. 3.1).

Figure 4.12: Upper left: Horizontally averaged logarithmic temperature as a function of height in the simulation box. The bars correspond to the scatter of the values in the horizontal layers. The solid lines represent time-averages of the total time-series. Upper right: Same as upper left, but for particle number density. Bottom left: Horizontally and temporally averaged vertical velocity as a function of height. Bottom right: Spatial averages of the absolute value of the magnetic field strength $|B|$ at each height for timestep $t=0$ min. Dotted lines indicate the corresponding minimum and maximum of $|B|$ at the respective height.
4.2 The Model Atmosphere

4.2.2 The Plasma-$\beta$ Term and the Heating Term

Fig. 4.13 shows histograms of the logarithmic plasma-$\beta$, the logarithmic kinetic plasma-$\beta$ and the logarithmic heating rate (=\(\eta\mu_j^2\)) as a function of height at \(t=40\) min. In addition, the heating scale height derived from the average heating rate of the entire time-series is shown. The atmosphere above 5 Mm is found to be in a low-$\beta$ state (\(\beta < 1\)) on average (Fig. 4.13, upper left). The comparison of $\beta$ and $\beta_{\text{kin}}$ illustrates that on average plasma-$\beta$ is 2-3 orders of magnitude larger than kinetic plasma-$\beta$ at each height (upper right). The heating rate shows an exponential decrease between approximately 6 and 28 Mm depending on the timestep chosen (Fig. 4.13, bottom row). The heating scale height obtained through a linear fit to the temporally averaged heating rate between 11 and 21 Mm is $H = 4.66 \pm 1.35$ Mm. It is comparable to the heating scale height of $H = 4.1$ Mm found for the active region part of the high-resolution time-series in the previous chapter (compare section 3.1.2).

Figure 4.13: Histograms of $\log \beta$ (upper left), $\log \beta_{\text{kin}}$ (upper right) and $\log \eta\mu_j^2$ (lower left) as a function of height for \(t=40\) min. Dashed lines correspond to the respective mean values. The $\log \beta$ and $\log \beta_{\text{kin}}$ maps are plotted on a logarithmic grayscale, the map for the heating rates is plotted on a linear grayscale. For a better comparison the mean value of $\beta$ is also plotted as a red line in the $\log \beta_{\text{kin}}$ plot. Lower right: Average heating rate of the entire time-series. The dotted lines indicate the average minimum and average maximum values of the time-series at each height, the solid line indicates the linear fit the heating rate.
4.2.3 Emissivities of the Extreme Ultraviolet Emission Lines

As discussed in section 2.4.3, the calculation of the emissivity for optically thin extreme ultraviolet emission lines is straightforward when ionization equilibrium is assumed. The emissivities of a set of lines (Table 2.4) is calculated. The time-dependent evolution of the O\textsubscript{vi} (1032 Å) emissivities has already been discussed in section 4.1. Fig. 4.1 shows the emissivity of the O\textsubscript{vi} (1032 Å) line integrated along the x-, y- and z-direction during the eruption. For comparison, the emissivities of Si\textsubscript{ii} (1533 Å), C\textsubscript{iv} (1548 Å), Ne\textsubscript{viii} (770 Å) and Mg\textsubscript{x} (625 Å) are shown in Fig. 4.1 for a single timestep \( t = 48 \) min of the eruption. The morphology of the resulting structures in the transition region lines is similar to the one discussed in section 3.2.1 where cool and dense low-lying structures are observed in e.g. the Si\textsubscript{ii} (1534 Å) and C\textsubscript{iv} (1548 Å). Lines formed in the high transition region and corona, e.g. Ne\textsubscript{viii} (770 Å) and Mg\textsubscript{x} (625 Å) show less structure and a smoother appearance than the coronal intensity maps of the previous time-series (see Fig. 3.6). The reason for this is most likely the different treatment of the magnetic field at the bottom boundary. The viscosity applied in this simulation run is higher than in the previous time-series which results in a faster damping of the velocities and leads to a smoother structure. As discussed in section 4.1.2, the blob appears bright in emission above the limb in the Si\textsubscript{ii} (1533 Å), C\textsubscript{iv} (1548 Å) and Ne\textsubscript{viii} (770 Å) lines and mostly dark when viewed against the background emission in the Mg\textsubscript{x} (625 Å) line except for the outer edge (Fig. 4.1). This also shows that the temperature inside the blob is below the line formation temperature of the Mg\textsubscript{x} (625 Å) line (\( T_f \approx 10^6 \) K, see sections 4.1.2 and 4.1.3).

\[ \text{Figure 4.14: Emissivities of Si\textsubscript{ii} (1533 Å), C\textsubscript{iv} (1548 Å), Ne\textsubscript{viii} (770 Å) and Mg\textsubscript{x} (625 Å) integrated along the y-axis at } t = 48 \text{ min. The images are plotted on a logarithmic scale relative to the mean value of the integrated emissivity ranging from 0.1 to 10. The different line formation temperatures are indicated in the lower right of each panel.} \]

4.2.4 Investigation of the Doppler Shift Maps

In the last column of Fig. 4.1, Doppler maps of the O\textsubscript{vi} (1032 Å) line are shown for a line-of-sight integration along the vertical direction. The signatures of
4.2 The Model Atmosphere

the blob are clearly visible. The blob appears blue indicating plasma upflows during its rising phase, between $t=39$ min and $t=44$ min. The blob appears red indicating downflows during the phase where it descends, between $t=48$ min and $t=54$ min. At $t=46$ min half of the blob is still blueshifted, while the other half of the blob already shows a downflow observed in the form of a redshift (Fig. 4.1). In addition, Fig. 4.15 shows Doppler maps of various emission lines that are formed between $T = 2.5 \times 10^4$ K and $T = 10^6$ K. As the line formation temperature increases, an increase of blueshift is observed in the Doppler maps. Also the appearance of the blob changes slightly when being viewed in different emission lines, which is a consequence of both the time-dependent evolution of the blob (see sections 4.1.2 and 4.1.3) and the height-dependence of the various Doppler maps.

![Figure 4.15: Doppler maps of Si II (1533 Å), C III (977 Å), C IV (1548 Å), O IV (1401 Å), O V (630 Å), O VI (1032 Å), Ne VIII (770 Å) and Mg X (625 Å) at $t=48$ min. The line formation temperatures are indicated in the lower right of each plot. All maps are scaled from -10 km/s (blue) to +10 km/s (red).](image)

4.2.5 The Heating Structure

In Fig. 4.16 the current density squared $j^2$ is shown integrated through the box along the $y$-direction. $j^2$-images for the complete time-series at a cadence of 5 minutes are presented. Sites of increased heating are indicated by bright features in the $j^2$-plots.
Figure 4.16: Time-series of $j^2$ images integrated along the y-axis. The images have been divided by the mean values of the t=0 min image at each height. The scaling is linear from 0 to 0.015. The red box corresponds to the region where the blob starts to form (see section 4.1.1).

The filamentary structure of the heating layers at all timesteps shows that the heating is directed along the magnetic field. Increased heating starts in a long stripe above the actual blob formation region at approximately t=15 min. At t=20 min heating starts in the formation region of the blob as well. In the course of the
simulation the heating increases in the blob formation region and eventually causes the onset of the plasma eruption as shown in section 4.1.1. The integrated current density squared reaches its largest value shortly before the plasma blob is ejected at approximately $t=40$ min. After the ejection of the plasma blob the heating is slowly fading away (Fig. 4.16 $t=45-70$ min). At $t=50$ min increased heating is seen at the front of the blob which results from the deformation of the magnetic field by the blob as described in section 4.1.3.4.
5 Discussion

Results of extreme ultraviolet emission line spectra synthesized from three dimensional magnetohydrodynamic models of the solar corona have been presented in chapter 3 and 4. Emission lines formed in the thin transition region between the chromosphere and the corona have been used to study the structure and dynamics of the lower corona and the transition region to the subjacent chromosphere. These lines are most common in the extreme ultraviolet region (500 - 2000 Å) of the spectrum. The profiles of these optically thin lines provide information on mass motions that transfer energy into the corona. These processes result in observable spectral signatures such as Doppler line shifts and non-thermal line broadening.

Whereas the transition region is highly structured and extremely dynamic, there are two characteristics deviated from observed spectra that are known to be invariant in terms of space and time; the shape of the emission measure curve (section 2.3.2.1) and the average Doppler shifts of spectral lines (section 1.1.3.2). These characteristics are therefore suitable to be examined with the model. As shown in sections 3.2.3 and 3.3.3, the Doppler shifts and emission measure function observed are reproduced by the model. These results indicate that the above characteristics that are based on observed spectra, are caused by a heating mechanism as it is assumed in the model, i.e. heating caused by braiding of magnetic field lines resulting from the photospheric plasma motions.

Emission Line Synthesis  The intensity maps of various spectral lines of both the transition region and the corona as synthesized from the model (section 3.2.1) show substantial similarity to images of the corona recorded by e.g. the TRACE satellite in full-size active regions (Fig. 1.0) or SUMER raster scans. In the simulation box, as a result of the efficient heat conduction at high temperatures, the coronal emission appears diffuse and to continuously extend from the lower transition region to the corona. In contrast, cool, dense low-lying loops are observed in the transition region. It should be mentioned though that it cannot be excluded that some of the loops are the result of line-of-sight effects. This is indicated in Fig. 3.7 where some loops appear and disappear while the simulation box is rotated around the z-axis. As can be seen from the timescales of the intensity variations in the model.
the corona reacts more slowly to changes in the heating rate than the transition region, in accordance to observations. This is particularly obvious when the sequence of intensity maps is run as a movie. Variations on the time-scale of minutes are seen in the intensity maps of both C\textit{iv} (1548 Å) and Ne\textit{viii} (770 Å), with C\textit{iv} varying stronger. This phenomenon is a result of the large differences in the cooling time-scales of the transition region (in the order of seconds) and of the corona (about one hour).

**Doppler shifts** Before discussing results on the Doppler shifts, a principal note should be made. The average Doppler shift depends on the spatial resolution of the spectral maps considered as can be seen from the results presented in section 3.2.3.2. If initially, all gridpoint spectra of the box are averaged horizontally and subsequently, the shift of the resulting spectrum is determined (shift average spectrum), the Doppler shifts of transition region lines will be up to 4 km/s larger than the mean Doppler shift calculated directly from the Doppler maps (mean shift). Also, the maximum of the Doppler shift-temperature curve will be shifted to higher temperatures. It must therefore be ensured that the same calculation method will be used when comparing the average values of synthetic and observed spectra.

The Doppler maps of the transition region lines are found to be highly structured compared to Doppler shift maps of coronal lines. The absolute amplitude of line shifts in the Doppler maps of the transition region lines is smaller than that of the coronal lines (section 3.2.3.2). The match of the average Doppler shifts of both observation and MHD model is remarkable in the transition region up to line formation temperatures of log$T=5.5$. The difference between modeled and observed Doppler shift is less than 1σ for most lines in the transition region. Regarding the coronal Doppler shifts, the model shows the expected decrease of line shifts at higher temperatures. On the other hand, the observed Ne\textit{viii} (770 Å) and Mg\textit{x} (625 Å) blueshifts are not reproduced by the MHD model. The discrepancy between both observed Doppler shifts and model Doppler shifts of coronal lines is also apparent when Doppler shift distributions are evaluated (Fig. 3.12). Whereas the Doppler shift distributions of transition region lines of both observation and model peak at roughly the same velocities, the small fraction of blueshifts in the asymmetric distributions of coronal lines leads to positive median values, i.e. redshifts, in the model. Differences in the width of the coronal Doppler shift distributions of the model and the observation are not seen. The inability of the MHD model to reproduce the observed blueshifts of coronal emission lines is a well-known problem assumed to be due to the influence of the impenetrable upper boundary condition of the model. This condition is expected to damp flows along magnetic field lines that intersect the
upper boundary (Peter et al., 2004). New models with an upper boundary shifted higher up into the corona will reduce such boundary effects and are supposed to provide an even better match to the observed Doppler shifts.

**Differential Emission Measure** Since the Skylab mission in the 1970s, loops have been recognized to be an essential component of the solar corona playing an important role in the energy balance. In the present literature, the corona is even supposed to be entirely composed of nested loops of various lengths, temperatures and heating rates. A nested structure of low-lying cool dense loops was first suggested by Dowdy et al. (1986) in order to explain the temperature dependence of the emission measure function that describes the emission efficiency of the plasma in a given temperature range. The observed distribution of emission measure over temperature predicts a rise of the differential emission measure (DEM) function for temperatures below $T = 10^5$ K. Whereas early models of the transition region failed to predict such a rise, the DEM results obtained from our MHD model show substantial similarity to the overall shape of the observed DEM distribution (section 3.3). In particular, the rise of the DEM distribution at temperatures below $T = 10^5$ K is pictured by the model. The DEM distribution of the model has a local minimum near $\log T = 5.2$ (compared to $\log T = 5.5$ in the observation) and reproduces the observed turnover of the DEM curve at about one million Kelvin. This is remarkable since no fine-tuning has been applied to the model.

**Mass Flux Balance** Observed and modeled downflow velocities in the transition region range from 2 km/s to 10 km/s. The following estimates imply that these downflows can deplete the corona within 500 s when the assumption is made that the total particle flux leaving the corona is $f = \gamma \cdot v \cdot n \cdot A$ (Mariska, 1992). The filling factor of the observed downflows is assumed to be $\gamma = 0.15$, $v$ is assumed to be 8 km/s according to the observed Doppler velocity of C iv (1548 Å), $n = 10^{16}$ m$^{-3}$ is a typical transition region density, and $A$ is the surface area of the Sun. The total particle flux is found to be $f \approx 8 \times 10^{37}$ s$^{-1}$. The total number of particles in the corona is $N = A_c \cdot h_c \cdot n_c \approx 4 \times 10^{40}$, where $A_c$ is the surface of the coronal sphere, $h_c = 50,000$ km is the coronal scale height, and $n_c = 10^{14}$ m$^{-3}$ is the coronal density. These simplified assumptions result in the aforementioned timescale of approximately $\tau \approx 500$ s during which the entire corona would drain.
To replenish the corona which loses mass by downflows, spicule-associated upflows are assumed to play a significant role, e.g. (McIntosh and Pontieu, 2009). As found in this study and discussed above, the model does not show considerable blueshifts in the coronal lines that would indicate upflows. As a consequence, one would expect that the coronal mass of the model decreases. However, the present findings do not indicate such a depletion (section 3.4.2), but rather indicate a constant coronal mass - over a time period of 30 minutes which is significantly longer than the calculated depletion time. The finding of a constant coronal mass in the model also provides evidence that the dominating redshifts of the model are not due to an artifact caused by the initial conditions of an overdense corona, where the excess mass is simply falling down.

Nevertheless, the discrepancy remains open that the coronal mass is balanced during the simulation period, whereas at the same time only mass downflows are being observed at the examined isosurface temperature layers in the box. However, this discrepancy is mitigated by the finding of section 3.4.2 that the transition region redshifts are overestimated when interpreted exclusively as mass downflows. Also, some weaker blueshifts can be seen in the Doppler shift maps of both transition region and coronal lines. Possible reasons for upflows not having been observed and resolved satisfactorily so far, are (1) the simplified assumption of a one-dimensionally stratified atmosphere when isosurfaces of constant temperature are considered, whereas the intensity images of the C IV (1548 Å) and Ne VIII (770 Å) lines in Fig. 3.6 suggest a complex temperature structure of the solar atmosphere, and that (2) vertical velocities and mass fluxes only have been considered in this study, whereas it must be assumed that fluxes parallel to the corrugated isosurfaces would also contribute to the total mass balance of the corona.

The previous discussion shows that the model in principal reflects the coronal dynamics. However, it remains desirable to consider more in depth possible upflow mechanisms of the model, like it is suggested in the following.

**Chromosphere-Corona Mass Cycle** The analysis of mass flows along magnetic field lines near a small X-type reconnection site suggests a new scenario of a chromosphere-corona mass-cycle (section 3.5.3). The field lines are intermittently connected to sites of increased heating where plasma is being injected into the loop (observed as blueshifts), and to sites with less heating where the plasma drains from the loop (observed as redshifts). The observed blueshifts and redshifts are a consequence of the continuously changing three-dimensional magnetic structure, an inherent three-dimensional process. The question if this is a ubiquitous feature remains unsolved up to now, and further model calculations will be needed.
**Transient Ejection**  The formation and subsequent evolution of a plasma ejection has been investigated in the second part of this work (chapter 4). The ejection in the form of a plasma blob appears to be a hydrodynamic effect rather than a phenomenon associated with the changing magnetic field, e.g. a jet. The inspection of the vertical velocity at the start of the ejection, i.e. \( v_z \approx 50 \text{ km/s} \), shows that the velocity of the plasma blob is of the order of the sound speed (\( c_s \approx 50 \text{ km/s} \) at \( T = 10^5 \text{ K} \)) and corresponds to only one sixth of the Alfvén speed (\( v_A \approx 300 \text{ km/s} \) at magnetic field strengths of \( B = 1 \text{ G} \) and particle densities of \( n = 10^{14} \text{ m}^{-3} \)) in the corona.

Prior to the rise of the blob the formation of a soliton-like structure is observed in the simulation box (section 4.1.1). Enhanced heating causes a strong temperature increase in the formation region of this structure and eventually leads to the onset of the eruption (section 4.1.4). Subsequently, the plasma blob expands and its average temperature decreases (section 4.1.2). This decrease in temperature is found to be stronger than expected for an adiabatic expansion (section 4.1.3). During the eleven minutes flight time the mass of the plasma blob increases by more than a factor of two. It is shown that this mass increase can be accounted for by the mass that is flowing into the tube as the blob starts to rise (section 4.1.5). The mass increase of the blob might therefore be due to the method applied to characterize it. The blob rises parallel to the magnetic field lines, and while expanding changes its shape from tailed to ellipsoidal. As it descends, the blob deforms the magnetic field dynamically. As a consequence, it is slowed down and heated at the front. The above projection of the plasma blob by the MHD model is a surprising phenomenon. It is desirable to confirm this predicted phenomenon by observable data, for example image-based procedures or derivation from spectral data.
A Probability Density Function

A more general form of the Gaussian-shaped line profile function (Eq. 2.73) derived in section 2.4.4 is

\[ f(x) = \frac{\epsilon}{\sqrt{\pi} \sigma} \exp \left[ -\frac{(x - \mu)^2}{\sigma^2} \right], \quad (A.1) \]

where \( \epsilon \) is the emissivity of the respective emission line that is being considered. In this notation

\[ \int_{-\infty}^{\infty} f(x) \, dx = \epsilon. \quad (A.2) \]

The mean \( \mu \) and the standard deviation \( \sigma \) are given by

\[ \mu = \frac{1}{\epsilon} \int_{-\infty}^{\infty} x f(x) \, dx. \quad (A.3) \]

and

\[ \sigma = \left[ \frac{2}{\epsilon} \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx \right]^{1/2}. \quad (A.4) \]

The probability density function of the normal distribution with mean \( \mu \) and variance \( \sigma^2 \) is

\[ f(x; \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right]. \quad (A.5) \]

The mean \( \mu \) and the standard deviation \( \sigma \) are defined as

\[ \mu = \bar{x} = \int_{-\infty}^{+\infty} x f(x) \, dx \quad (A.6) \]

and

\[ \sigma = \left[ \int_{-\infty}^{+\infty} (x - \mu)^2 f(x) \, dx \right]^{1/2}. \quad (A.7) \]

For \( \mu = 0 \) and \( \sigma = 1 \) the probability density function reduces to

\[ f(x) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{x^2}{2} \right]. \quad (A.8) \]

In general, the density function is symmetric about its mean and the full-width-at-half-maximum (FWHM) and the standard deviation values are related through

\[ \text{FWHM} = 2\sqrt{\ln 2} \cdot \sigma \approx 2.3548 \sigma. \quad (A.9) \]
B The Viscous Force

The identity of the expressions \[2.29\] and \[2.31\] for the viscous force is shown (section \[2.2.2.4\]). Applying the product rule to the right-hand-side of Eq. \[2.29\] yields

\[
F_\nu = 2\nu\rho\nabla S + 2\rho\nu S\nabla \ln \rho. \tag{B.1}
\]

Therefore, it has to be shown that

\[
\nabla S = \frac{1}{2} \left( \nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) \right), \tag{B.2}
\]

where

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{v} \right). \tag{B.3}
\]

The identity (Eq. \[B.2\]) is shown exemplarily for the first component of \(\nabla S\):

\[
[\nabla S]_1 = \left. \frac{\partial}{\partial x_1} \left( \frac{\partial v_1}{\partial x_1} - \frac{1}{3} \nabla \cdot \mathbf{v} \right) \right|_{1} + \left. \frac{\partial}{\partial x_2} \left( \frac{1}{2} \left( \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} \right) \right) \right|_{1}
\]

\[
= \left. \frac{\partial v_1^2}{\partial x_1} - \frac{1}{3} \frac{\partial}{\partial x_1} (\nabla \cdot \mathbf{v}) + \frac{1}{2} \frac{\partial^2 v_2}{\partial x_2 \partial x_1} + \frac{1}{2} \frac{\partial^2 v_1}{\partial x_2 \partial x_1} + \frac{1}{2} \frac{\partial^2 v_3}{\partial x_3 \partial x_1} + \frac{1}{2} \frac{\partial^2 v_1}{\partial x_3 \partial x_1} \right|_{1}
\]

\[
= \frac{1}{2} \left( \frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_2}{\partial x_2^2} + \frac{\partial^2 v_3}{\partial x_3^2} \right) + \frac{1}{2} \left( \frac{\partial^2 v_1}{\partial x_2 \partial x_1} + \frac{\partial^2 v_2}{\partial x_2 \partial x_1} + \frac{\partial^2 v_3}{\partial x_3 \partial x_1} \right) - \frac{1}{3} \left( \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + \frac{\partial^2 v_3}{\partial x_1 \partial x_3} \right)
\]

\[
= \frac{1}{2} \left( \frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_2}{\partial x_2^2} + \frac{\partial^2 v_3}{\partial x_3^2} \right) + \frac{1}{2} \frac{1}{3} \left( \frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + \frac{\partial^2 v_3}{\partial x_1 \partial x_3} \right) \right|_{1}
\]

\[
= \frac{1}{2} \left( \nabla^2 \mathbf{v} \right)_1 + \frac{1}{3} \left( \nabla (\nabla \cdot \mathbf{v}) \right)_1 \quad \text{q.e.d.} \tag{B.4}
\]
C Evaluation of the Mean Molecular Weight

To evaluate the atomic weight $\mu$ in the solar corona, one must determine the average particle mass $\bar{m}$ by adding over all particles of the solar corona

$$\bar{m} = \frac{\sum_{j} n_j m_j + n_e m_e}{\sum_{j} n_j + n_e} \approx \frac{\sum_{j} n_j m_j}{\sum_{j} n_j + n_e}, \quad (C.1)$$

where $n_j$ is the number density of atoms of type $j$, $m_j$ is the corresponding mass, and $n_e$ is the number density of electrons. The electron mass is assumed to be negligible compared to the proton mass ($m_e/m_p = 1835$). The mass of the atom of type $j$ is determined from the mass number $A_j$ (number of protons and neutrons),

$$m_j \approx A_j m_H, \quad (C.2)$$

where the binding mass and relativistic effects have been neglected. The mean atomic weight can be written

$$\mu = \frac{\bar{m}}{m_H} = \frac{\sum_{j} n_j A_j}{\sum_{j} n_j + n_e} \quad (C.3)$$

For fully ionized gas the number density of electrons is $n_e = \sum_{j} n_j Z_j$, where $Z_j$ is the atomic number. Therefore, the mean atomic weight can be expressed as

$$\mu = \frac{\sum_{j} n_j A_j}{\sum_{j} n_j (1 + Z_j)} \quad (C.4)$$

The number density $n_j$ depends on the mass fractions $X_j$ of elements in the region considered

$$n_j = \frac{\rho}{m_H A_j} X_j \quad (C.5)$$

The mass fractions of hydrogen, helium and heavier elements in the corona are $X = 0.70, Y = 0.28$ and $Z = 0.02$. The relation $\sum_{j} X_j = 1$ always holds. For a fully ionized gas the mean molecular weight is

$$\mu = \left[ \sum_{j} \frac{X_j (1 + Z_j)}{A_j} \right]^{-1} \approx \left[ 2X + \frac{3Y}{4} + \frac{Z}{2} \right]^{-1}, \quad (C.6)$$
where for elements heavier than helium it is assumed that \((1 + Z_j)/A_j \approx 1/2\). Inserting the coronal mass fractions of hydrogen, helium and heavier elements into \([C.6]\) the mean molecular weight for the coronal plasma is \(\mu = 0.6\).
Bibliography


J. Clyne and M. Rast. A prototype discovery environment for analyzing and visualizing terascale turbulent fluid flow simulations. *Proceedings of Visualization and Data Analysis*, 5669:284, 2005. 3.7, 5.5.1


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Bibliography


K. P. Dere, E. Landi, H. E. Mason, B. C. Monsignori Fossi, and P. R. Young. CHIANTI - An atomic database for emission lines - I: Wavelengths greater than 50 Å. *Astronomy and Astrophysics Supplement Series*, 125:149–173, 1997. 2.3.2.4


P. V. Foukal. *Solar Astrophysics*. VILEY-VCH Verlag GmbH, 2004. 2.2.2.4


Bibliography

P. Mazzotta, G. Mazzitelli, S. Colafrancesco, and N. Vittorio. Ionization balance for optically thin plasmas: Rate coefficients for all atoms and ions of the elements H to Ni. *Astronomy and Astrophysics Supplement Series*, 133:403–409, 1998. 2.3.2.3


S. W. McIntosh, J. C. Brown, and P. G. Judge. The relation between line ratio and emission measure analyses. *Astronomy and Astrophysics*, 333:333, 1998. 3.3.1


H. Peter. On the nature of the transition region from the chromosphere to the corona of the sun. *Astronomy and Astrophysics*, 374:1108, 2001. 3.2.3 3.17


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